Even-numbered solutions

2. Limit compare to harmonic series.

$$\lim_{n\to\infty}\left|\frac{n-1}{n^2+n}\cdot\frac{n}{1}\right| = 1,$$ so the series converges.

4. This is an alternating series with $a_n = \frac{n-1}{n^2+n}$.

Since the denominator grows faster than the numerator we have $a_n > a_{n+1}$ and $\lim_{n\to\infty} a_n = 0$. Hence the series converges by AST. Note that by #2 this is only conditional convergence.

6. $$\frac{1}{n+n\cos^2(n)} \geq \frac{1}{n+n} = \frac{1}{2n}$$ so the series

diverges by comparison to the harmonic series.

8. Try ratio test:

$$\lim_{n\to\infty}\left|\frac{2^{k+1}(k+1)!}{(k+3)!}\cdot\frac{(k+2)!}{2^k\cdot k!}\right|$$

$$=\lim_{n\to\infty}\left|\frac{2(k+1)}{(k+3)}\right| = 2 > 1$$ so the series diverges.

Alternatively look at $\frac{2^k k!}{(k+2)!} = \frac{2^k}{(k+1)(k+2)}$ ... diverges

by test for divergence: $\lim_{n\to\infty}\frac{2^k}{(k+1)(k+2)} = \infty \neq 0$.

10. Ratio test: $$\lim_{n\to\infty}\left|\frac{(n+1)^2}{e^{(n+1)^3}}\cdot\frac{e^{n^3}}{n^2}\right| = \lim_{n\to\infty}\left|\left(\frac{n+1}{n}\right)^2\cdot\frac{1}{e^{3n^2+3n+1}}\right|$$

$= 0 < 1$ so the

series converges absolutely.

12. Alternating series with $a_n = \frac{n}{n^2+25}$. Since n^2+25

grows faster than n, $a_n > a_{n+1}$ and $\lim_{n\to\infty} a_n = 0$. Thus the series converges by AST. It does not converge absolutely, however, since $\sum_{n=1}^{\infty}\frac{n}{n^2+25}$ diverges

by limit comparison to the harmonic series.

14. $\lim_{n\to\infty} \sin(n)$ DNE, so the series diverges by the test for divergence.

16. Limit compare to harmonic series.

$$\lim_{n\to\infty} \left| \frac{n^2+1}{n^3+1} \cdot \frac{n}{1} \right| = 1$$ so both series diverge together.

18. Alternating series with $a_n = \frac{1}{\sqrt{n}-1} > \frac{1}{\sqrt{n+1}-1} = a_{n+1}$

and $\lim_{n\to\infty} \frac{1}{\sqrt{n}-1} = 0$; so by AST the

series converges. Note that this is not absolute convergence since $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ diverges

by comparison to a p-series: $\frac{1}{\sqrt{n}-1} > \frac{1}{\sqrt{n}}$.

20. Use ratio test: $\lim_{k\to\infty} \left| \frac{k+6}{5^{k+1}} \cdot \frac{5^k}{k+5} \right|$

$= \lim_{k\to\infty} \left| \frac{k+6}{5(k+5)} \right| = \frac{1}{5} < 1$ so the series

converges absolutely.

22. Compare to p-series, $p=2$.

$$\frac{\sqrt{n^2-1}}{n^3+2n^2+5} \le \frac{n}{n^3+2n^2+5} \le \frac{n}{n^3} = \frac{1}{n^2}.$$

So the series converges.

24. Cosine oscillates so we must look at this in absolute values. $\left| \frac{\cos(\frac{n}{2})}{n^2+4n} \right| \le \frac{1}{n^2+4n} \le \frac{1}{n^2}$, so the

series converges absolutely by comparison to a p-series, $p=2$.

26. Ratio test: $\lim_{n\to\infty} \left| \frac{(n+1)^2+1}{5^{n+1}} \cdot \frac{5^n}{n^2+1} \right|$

$$= \lim_{n\to\infty} \left| \frac{n^2+2n+2}{5(n^2+1)} \right| = \frac{1}{5} < 1$$ so the

series converges absolutely.

28. Limit compare to p-series, $p=2$.

$$\lim_{n\to\infty} \left| \frac{e^{1/n}}{n^2} \cdot \frac{n^2}{1} \right| = \lim_{n\to\infty} e^{1/n} = 1,$$ so the

series converge together.

30. This is an alternating series with $a_j = \frac{\sqrt{j}}{j+5}$.

Since $j+5$ grows faster than $\sqrt{j}$, the terms are decreasing to zero. Then by AST, the series converges. Note that this is only conditional convergence, as $\sum_{n=1}^{\infty} \frac{\sqrt{j}}{j+5}$ diverges

by limit comparison to a p-series, $p=\frac{1}{2}$.

32. Trick: $\ln(n)^{\ln(n)} = \left(e^{\ln(\ln(n))}\right)^{\ln(n)} = \left(e^{\ln(n)}\right)^{\ln(\ln(n))} = n^{\ln(\ln(n))}$.

For large n, $\ln(\ln(n)) > 2$, so

$$\frac{1}{\ln(n)^{\ln(n)}} = \frac{1}{n^{\ln(\ln(n))}} < \frac{1}{n^2},$$ and so

the series converges by comparison to p-series $p=2$.

34. Limit compare to the harmonic series.

$$\lim_{n\to\infty} \left| \frac{2^{1/n}-1}{1/n} \right| = \lim_{x\to\infty} \left| \frac{2^{1/x}-1}{1/x} \right| \overset{LH}{=} \lim_{x\to\infty} \left| \frac{2^{1/x}\ln(2)\cdot(-\frac{1}{x^2})}{-\frac{1}{x^2}} \right|$$

$$= \lim_{x\to\infty} \left| 2^{1/x} \ln(2) \right| = \ln(2),$$

so the series diverge together.