MA 114 Worksheet # 22: Differential Equations and $y' = k(y - b)$

1. Conceptual Understanding:

   (a) What does it mean to say that a differential equation is first-order (or second-order or third-order...)

   (b) What does it mean to say that a differential equation is linear or nonlinear?

2. Use Separation of Variables to find the general solutions to the following differential equations.

   (a) $y' + 4xy^2 = 0$

   (b) $\sqrt{1-x^2}y' = xy$

   (c) $(1 + x^2)y' = x^3y$

   (d) $\sqrt{1+y^2}\,y' + \sec x = 0$

3. Solve $y' = 4y + 24$ subject to the condition that $y(0) = 5$.

4. Solve $y' + 6y = 12$ subject to the condition that $y(2) = 10$.

5. Recall that Newton's law of Cooling stipulates that the temperature $y(t)$ of a cooling object with respect to time satifies the differential equation

$$y' = -k(y - T_0),$$

   where k is a constant depending on the object and T_0 is the the temperature of the ambient environment.

   Frank's car engine runs at 210°F. On a 70°F day, he turns off the ignition and notes that five minutes later, the engine has cooled to 160°F.

   (a) Find the cooling constant k.

   (b) When will the engine cool to 100°F?

6. A cup of coffee with cooling constant $k = 0.09\text{min}^{-1}$ is placed in a room of temperature 20°C.

   (a) How quickly is the coffee cooling when the tempurature is 80°C?

   (b) Use the linear approximation to estimate the change in temperature over the next 6 s when the temperature is 80°C.

   (c) If the coffee is initially served at 90°C, how long will it take to reach an optimal drinking temperature of 65°C?

7. (Extra) A tank has the shape of the parabola $y = x^2$ revolved about the y-axis. Water leaks from a hole of area $B = 0.0005\ m^2$ at the bottom of the tank. Let $y(t)$ be the water level at time t. How long does it take for the tank to empty if the initial water level is $y(0) = 1\ m$?