

MA 114 Worksheet # 4: Summing an Infinite Series

1. (Review) Compute the following sums

(a) $\sum_{n=1}^5 3n$

(b) $\sum_{k=3}^6 \left(\sin\left(\frac{\pi}{2} + \pi k\right) + 2k \right)$

2. Conceptual Understanding:

(a) What is a series?

(b) What is the difference between a sequence and a series?

(c) What does it mean that a series converges?

3. Write the following in summation notation:

(a) $\frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \dots$

(b) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

4. Calculate S_3 , S_4 , and S_5 and then find the sum of the telescoping series $S = \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right)$.

5. Use Theorem 3 of 10.2 (Divergence Test) to prove that the following two series diverge:

(a) $\sum_{n=1}^{\infty} \frac{n}{10n+12}$

(b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

6. Use the formula for the sum of a geometric series to find the sum or state that the series diverges and why:

(a) $\frac{1}{1} + \frac{1}{8} + \frac{1}{8^2} + \dots$

(b) $\sum_{n=0}^{\infty} \left(\frac{\pi}{e} \right)^n$.

(c) $5 - \frac{5}{4} + \frac{5}{4^2} - \frac{5}{4^3} + \dots$

(d) $\sum_{n=0}^{\infty} \frac{8+2^n}{5^n}$.