Math 114: Worksheet 6

Absolute and Conditional Convergence

1. Conceptual understanding:

(a) Let $a_n = n/3n+1$. Does $\{a_n\}$ converge? Does $\sum_{n=1}^{\infty} a_n$ converge?

$\{a_n\}$ converges to $1/3$, but $\sum_{n=1}^{\infty} a_n$ does not converge by the divergence test

(b) Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ where $\lim_{n\to\infty} a_n = 0$.

$\sum_{n=1}^{\infty} \frac{1}{n}$, $\lim_{n\to\infty} \frac{1}{n} = 0$, but $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

(c) Does there exist a convergent series $\sum_{n=1}^{\infty} a_n$ which satisfies $\lim_{n\to\infty} a_n \neq 0$? Explain.

No, otherwise the divergence test would not be true.

(d) When does a series converge absolutely? When does a series converge conditionally?

* The series Σa_n converges absolutely if $\Sigma |a_n|$ converges.
* An infinite series Σa_n converges conditionally if Σa_n converges, but $\Sigma |a_n|$ diverges.

(e) State the Leibniz test for alternating series.

Assume that $\{a_n\}$ is a positive sequence that is decreasing and converges to 0. Then the following alternating series converges $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$

2. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty} (-1)^{n+1}/\sqrt[5]{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/5}}$

This converges by the alternating series test, but $\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1}}{n^{1/5}}\right| = \sum_{n=1}^{\infty} \frac{1}{n^{1/5}}$ diverges by the p-test. So it converges conditionally.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$, this converges by the alternating series test. Now consider $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{\ln(n+1)}\right| = \sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$. We know by the comparison test, that $\ln(n) < n$, so $\sum_{n=1}^{\infty} \frac{1}{\ln(n)} \geq \sum_{n=1}^{\infty} \frac{1}{n}$, which diverges. So this series converges conditionally

(c) $\sum_{n=1}^{\infty} 13\cos(5)^{n-1} = 13\sum_{n=1}^{\infty} \cos(5)^{n-1}$, which is a geometric series with $|r| = |\cos(5)| < 1$. So this series converges absolutely.

3. (a) False, To prove that the series $\sum_{n=1}^{\infty} a_n$ diverges you should compute the limit $\lim_{n\to\infty} a_n$. If this limit is not zero, then the series diverges

(b) True

(c) True