MA 114 Worksheet # 9: Power Series & Taylor Series

1. Use term by term integration and the fact that $\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan(x)$ to derive a power series centered at $x = 0$ for the arctangent function.
   [HINT: $\dfrac{1}{1+x^2} = \dfrac{1}{1-(-x^2)}$.]

2. Use the same idea as above to give a series expression for $\ln(1+x)$, given that $\displaystyle\int \frac{dx}{1+x} = \ln(1+x)$. You will again want to manipulate the fraction $\dfrac{1}{1+x} = \dfrac{1}{1-(-x)}$ as above.

3. Write $(1+x^2)^{-2}$ as a power series.
   [HINT: Use term by term differentiation.]

4. Find the terms through degree 3 of the Maclaurin series of $f(x)$.

   (a) $f(x) = (1+x)^{1/4}$.

   (b) $f(x) = e^{\sin(x)}$.

5. Find the Taylor series centered at c and find the interval on which the expansion converges to f.

   (a) $f(x) = \dfrac{1}{x}$ at $c = 1$.

   (b) $f(x) = e^{3x}$ at $c = -1$.

   (c) $f(x) = x^3 + 3x - 1$ at $c = 0$.

   (d) $f(x) = x^3 + 3x - 1$ at $c = 2$.