MA 114 Worksheet # 10: Taylor series, areas and volumes

1. Conceptual Understanding:

   (a) Give the Taylor series expansion of a function f and its relation to the Maclaurin series. Can every function that is infinitely differentiable be represented by some Taylor series?

   (b) State Taylor's theorem. Does this apply to all functions that have a convergent Taylor series expansion?

   (c) Can a function have two power series representations with the centered at the same point? Explain.

2. Assume that each of the following functions has a power series expansion centered at a given point a. Find the Taylor series about a. Specify the radius of convergence for the series.

   (a) $f(x) = \dfrac{1}{x}$ $\qquad a = 2$

   (b) $f(x) = \cos(x)$ $\qquad a = \pi$

3. Derive the Maclaurin series for $\cos(x)$ directly, that is without differentiating the formula for $\sin(x)$. Prove that $\cos(x)$ is equal to the sum of its Maclaurin series for all x.

4. Assume that each of the following functions has a power series expansion centered at a given point a. Find the Taylor series centered at a. Specify the radius of convergence for the series.

   (a) $f(x) = \dfrac{1}{x}$ $\qquad a = 2$

   (b) $f(x) = \cos(x)$ $\qquad a = \pi$

5. For each of the following, sketch the region enclosed by the given curves and compute the enclosed area.

   (a) $y = 12 - x^2,\ y = x^2 - 6$

   (b) $y = \dfrac{1}{x},\quad y = x,\quad y = \dfrac{1}{4}x,\ x > 0$

6. Conceptual Understanding:

   (a) Give an expression for the volume of a solid with base at height a, top at height b and cross section that has area $A(y)$ at height y for $a \le y \le b$.

   (b) Find a solid of revolution with volume given by the integral

   $$\pi \int_0^1 (1 - y)\, dy.$$

7. Find the volume of the solid obtained by rotating the region R about the specified line. Sketch the region and a typical cross section. Sketch or describe the solid.

   (a) R is the planar region bounded by $y = 1 - x^2$ and $y = 0$; about the x-axis.

   (b) R is the planar region bounded by $x = 2\sqrt{y}$, $x = 0$ and $y = 9$; about the y-axis.

   (c) R is the planar region bounded by $y = e^{-x}$, $y = 1$, and $x = 2$; about $y = 2$

8. Use the method of problem (6a) to find the volume of a cone of height h and base that is a disk of radius b

   (a) if the axis of the cone is the y-axis.

   (b) if the axis of the cone is the x-axis.

   Draw a picture in each case.