MA 114 Worksheet # 11:Volumes and averages

1. Conceptual Understanding:

   (a) Consider the continuous function $f(x)$ on the interval $[a, b]$. Divide this interval into n subintervals and from each subinterval take a point x_i^*. Write down an expression for the arithmetic average of $f(x_1^*), f(x_2^*), ..., f(x_n^*)$. Show that as $n \to \infty$, you obtain the correct expression for the average value of f on $[a, b]$.

   (b) State the mean value theorem for integrals. Prove the theorem by applying the mean value theorem for derivatives to the function $F(x) = \int_a^x f(t)\, dt$.

   (c) If a solid has a cross-sectional area given by the function $A(x)$, what integral should be evaluated to find the volume of the solid?

2. Calculate the volume of the following solid. The base is a square, one of whose sides is the interval [0,l] along the x-axis. The cross-sections perpendicular to the x-axis are rectangles of height $f(x) = x^2$.

3. The base of a certain solid is the triangle with vertices at $(10, 5)$, $(5, 5)$, and the origin. Cross-sections perpendicular to the y-axis are squares. Find the volume of the solid.

4. As viewed from above, a swimming pool has the shape of the ellipse $\dfrac{x^2}{2500} + \dfrac{y^2}{1600} = 1$. The cross sections perpendicular to the ground and parallel to the y-axis are squares. Find the total volume of the pool.

5. Find the average value of each of the following functions on the specified interval.

   (a) $f(x) = x^2 + 1$ on $[1, 3]$

   (b) $g(x) = e^x$ on $[-1, 1]$

   (c) $h(x) = \dfrac{3}{x^2 + 1}$ on $[0, 1]$

6. Set up an integral for the volume of the solid obtained by rotating the region R about the specified line. Do not evaluate the integral. Sketch the region, a typical cross section, and the solid.

   (a) R is the region bounded by the curves $y = 1 - x^2$ and $y = 0$; about the line $y = -1$.

   (b) R is the region bounded by the curves $y = 1/x$, $x = 1$, $x = 2$ and $y = 0$; about the y-axis.

   (c) R is the region bounded by the curves $y = 3 + 2x - x^2$, $x + y = 3$; about the y-axis.