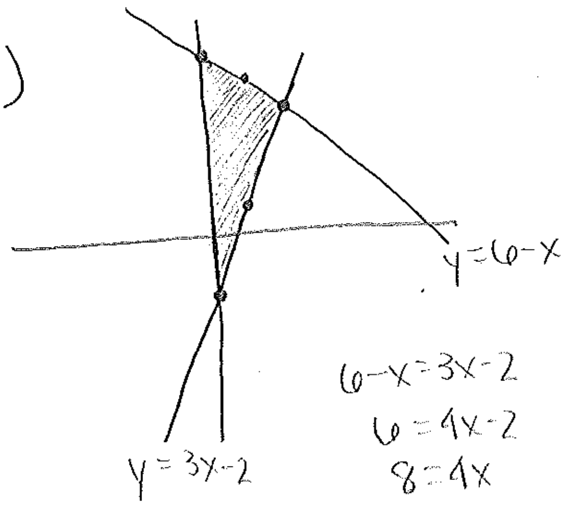
MA 114 Worksheet #12 Volumes of Revolution (Shell Method)

1) a) $V = 2\pi \int_a^b x f(x)\, dx$

b) Integrating with respect to y because you are rotating around the y-axis.

2) a)



$f(x) = 6 - x$

$g(x) = 3x - 2$

$V = 2\pi \int_0^2 x\left(6 - x - (3x - 2)\right) dx$

$6 - x = 3x - 2$

$6 = 4x - 2$

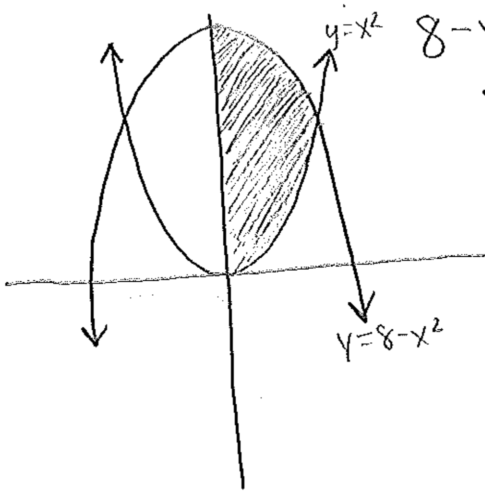
$8 = 4x$

$x = 2$

$V = 2\pi \int_0^2 x(8 - 4x)\, dx$

$V = 2\pi \int_0^2 [8x - 4x^2]\, dx$

b)



$8 - x^2 = x^2$

$8 = 2x^2$

$x^2 = 4$

$x = 2$

$V = 2\pi \int_0^2 x\left(8 - x^2 - x^2\right) dx$

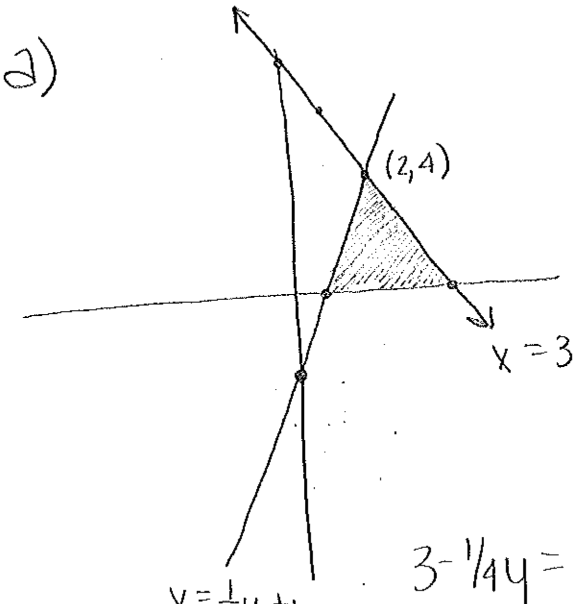
$= 2\pi \int_0^2 x(8 - 2x^2)\, dx$

$f(x) = 8 - x^2$

$g(x) = x^2$

$V = 2\pi \int_0^2 [8x - 2x^3]\, dx$

3) a)



$f(y) = 3 - 1/4y$

$g(y) = \frac{1}{4}y + 1$

$V = 2\pi \int_0^4 y\left[3 - 1/4y - \left(\frac{1}{4}y + 1\right)\right] dy$

$3 - 1/4y = 1/4y + 1$

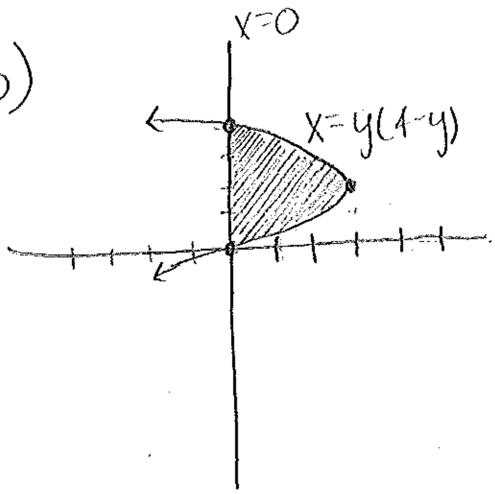
$3 = 1/2y + 1$

$1/2y = 2$

$y = 4$

$= 2\pi \int_0^4 y(2 - 1/2y)\, dy$

b)



$f(y) = y(4-y)$

$g(y) = 0$

$V = 2\pi \int_0^4 y\left[y(4-y)\right] dy$

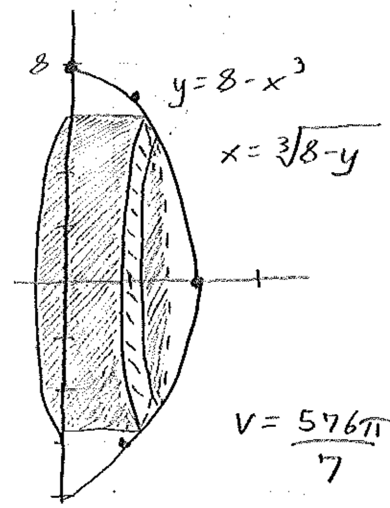
$= 2\pi \int_0^4 (4y^2 - y^3)\, dy$

$x = y(4-y)$

$x = 4y - y^2$

Worksheet 12 Key

4. $f(x) = 8 - x^3$ on $[0, 2]$



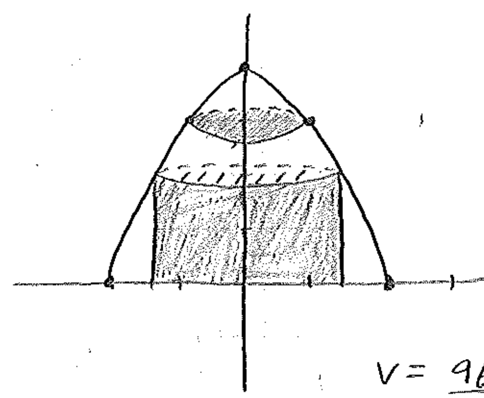
a) Disk method: Cross sections are circular disks with radius of $8 - x^3$ and area $A(x) = \pi(8 - x^3)^2$.

$$V = \int_0^2 \pi(8 - x^3)^2.$$

$$V = \frac{576\pi}{7}$$

Shell method: shells with radius y have height $\sqrt[3]{8-y}$, and surface area $A(y) = 2\pi y \sqrt[3]{8-y}$.

$$V = \int_0^8 2\pi y \sqrt[3]{8-y}\, dy$$



b) Disk method: Cross sections are circular disks with radius $\sqrt[3]{8-y}$ and area $A(y) = \pi(\sqrt[3]{8-y})^2$.

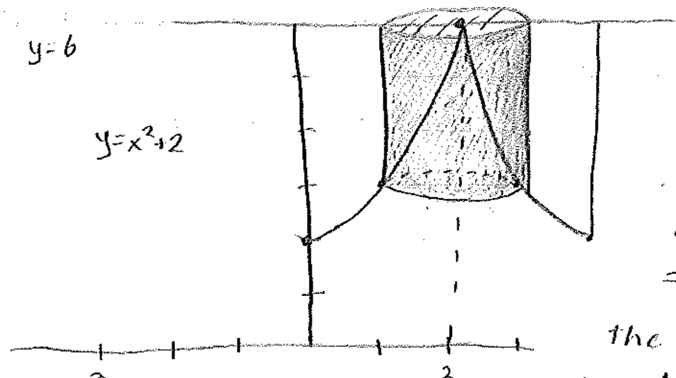
$$V = \int_0^8 \pi(\sqrt[3]{8-y})^2\, dy$$

$$V = \frac{96\pi}{5}$$

Shell method: Shells with radius x have height $8 - x^3$ and surface area $A(x) = 2\pi x(8 - x^3)$.

$$V = \int_0^2 2\pi x(8 - x^3)\, dx$$

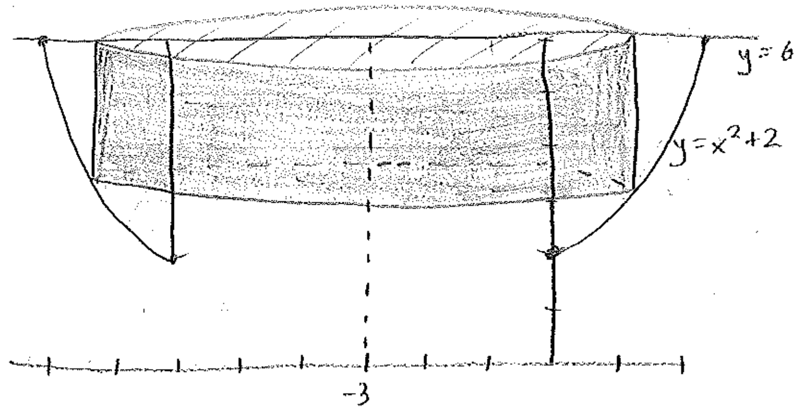
5. a)



Shells with left edge at x have radius $2-x$ and height $6-(x^2+2) = 4-x^2$. Thus the surface area is $A(x) = 2\pi(2-x)(4-x^2)$.

$$V = \int_0^2 2\pi(2-x)(4-x^2)\, dx = \frac{128\pi}{3}$$

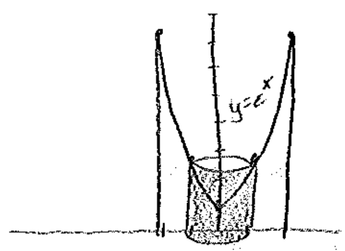
b)



Shells with right edge at x have radius $x-(-3)$ $= x+3$, height $6-(x^2+2) = 4-x^2$, and surface area $A(x) = 2\pi(x+3)(4-x^2)$.

$$V = \int_0^2 2\pi(x+3)(4-x^2)\,dx = 64\pi$$

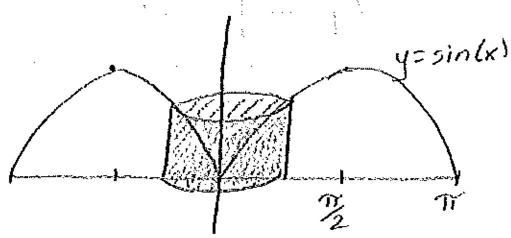
6. a)



Shells with radius x have height e^x and surface area $A(x) = 2\pi x e^x$.

$$V = \int_0^2 2\pi x e^x\,dx = 2\pi(1+e^2)$$

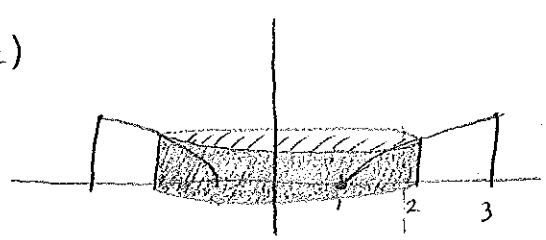
b)



Shells with radius x have height $\sin(x)$ and surface area $A(x) = 2\pi x\sin(x)$.

$$V = \int_0^{\pi} 2\pi x\sin(x)\,dx = 2\pi^2$$

c)



Shells with right edge at x have radius x, height $\ln(x)$, and surface area $A(x) = 2\pi x\ln(x)$.

$$V = \int_1^3 2\pi x\ln(x)\,dx = -4\pi + 9\pi\ln(3)$$