Worksheet 13 Key

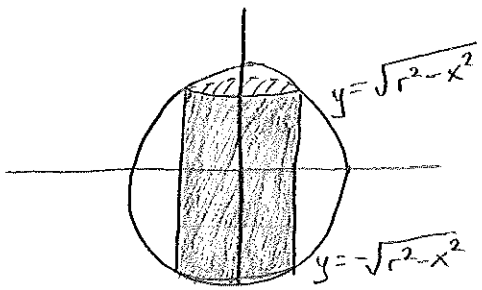
1. a) Work is a measurement (in joules) of the energy required to move an object. While work takes into account the effect on a mass, force doesn't.

b) A Newton is $N = Kg\,\frac{m}{s^2} = Kg \cdot g$, where g is acceleration due to gravity, $g \approx 9.8$. Then the force due to gravity exerted on a 1 Kg weight is $1Kg \cdot 9.8\ m/s^2 = 9.8\ N$. To lift the weight 1 m, $9.8\ N \cdot 1m = 9.8$ joules are required.

c) Since the weight is displaced 0 meters, the work done is $\int_0^0 9.8\,dh = 0$. Or we may see this from the work being equal and opposite in the opposite directions.

d) Rotate the right half of the circle $x^2+y^2=r^2$ about the y-axis. Shells of radius x have height $2\sqrt{r^2-x^2}$, and surface area

$$A(x) = 2\pi x \cdot 2\sqrt{r^2-x^2}$$



Then $V = \int_0^r 4x\pi\sqrt{r^2-x^2}\,dx$

Put $u = r^2-x^2$, Then $du = -2x\,dx$.

Then $V = \int_{r^2}^0 -2\pi\sqrt{u}\,du = \int_0^{r^2} 2\pi u^{1/2}\,du$

$$= \frac{2\pi}{3/2}\left[u^{3/2}\right]_0^{r^2} = \frac{4\pi r^3}{3}$$

MA 114 Worksheet 13 – Work

2) $F = ma = 500\,kg \cdot 9.8\,m/s^2$

$F(x) = 500 \cdot 9.8$

$$W_1 = \int_a^b F(x) = \int_0^{1000} 500 \cdot 9.8\,dx$$ (moving from bottom to top)

$$W_2 = \int_a^b F(x) = \int_{1000}^{0} 500 \cdot 9.8\,dx = -\int_0^{1000} 500 \cdot 9.8\,dx$$ (moving from top to bottom)

Round Trip: $W = \int_0^{1000} 500 \cdot 9.8\,dx - \int_0^{100} 500 \cdot 9.8\,dx = 0$

Worksheet #13

First find the force required at equilibrium

3. $k \cdot x = 15$, $k \cdot (.5) = 15 \Rightarrow k = 30 N/m$ at equilibrium

$\int_0^1 30x\,dx$

$= \frac{30}{2} x^2 \Big|_0^1$

$= 15 \cdot (1)^2 - 0 = 15$ J

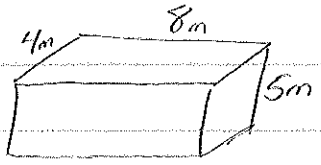
4. Since it stretches 10cm past its natural length $k \cdot x = F$

$k(.1) = 50$ N $\Rightarrow K = 500$N

From 15cm to 30cm stretches 15 from natural so:

$W = \int_{.1}^{.25} 500x\,dx = 250x^2 \Big|_{.1}^{.25}$

$= 250(.25)^2 - 250(.1)^2 = 13.125$ J

5. 

density of water is $1{,}000\ kg/m^3$

First, compute work against gravity:

$A(y)\,\Delta y$

Force on layer $= g \times \text{density} \times A(y)\Delta y$

$= 9.8(1{,}000)(8 \times 4)\,\Delta y$ ← or just y

Work on layer $= 9.8(1{,}000)(8 \times 4)\,\Delta y\,(5 - y)$

$= 313{,}600(5 - y) = 1{,}568{,}000 - 313{,}600y$

$\int_0^5 1{,}568{,}000 - 313{,}600y\,dy$

$1{,}568{,}000y - 156{,}800y^2 \Big|_0^5$

$1{,}568{,}000(5) - 156{,}800(5)^2 - 0 = 7{,}840{,}000 - 3{,}920{,}000 =$

$3{,}920{,}000$ J