MA 114 Worksheet # 14: Integration by Parts and by Trigonometry

1. Conceptual Understanding:

   (a) Give the integration by parts formula.

   (b) Use the product rule and the fundamental theorem of calculus to prove the definite integral version of the integration by parts formula. That is, prove:

   $$\int_a^b f(x)g'(x)dx = f(x)g(x)\,\Big|_a^b - \int_a^b g(x)f'(x)dx$$

   for functions f and g continuous on $[a, b]$ and differentiable on (a, b).

2. Evaluate the following integrals.

   (a) $\displaystyle\int \cos^2(x)\,dx$

   (b) $\displaystyle\int \ln t\,dt$

   (c) $\displaystyle\int_1^2 y^4 \ln y\,dy$

   (d) $\displaystyle\int e^x \cos(x)\,dx$

   (e) $\displaystyle\int_0^{\pi/2} \sin^2(x)\cos^2(x)\,dx$

   (f) $\displaystyle\int \cos(\ln x)\,dx$

   (g) $\displaystyle\int \sin^3(x)\cos^2(x)\,dx$

   (h) $\displaystyle\int x^2 \sin x\,dx$

   (i) $\displaystyle\int \frac{\sin\phi}{\cos^3\phi}\,d\phi$

   (j) $\displaystyle\int \tan^5(x)\sec^3(x)\,dx$

3. Prove that for m, n integers

   $$\frac{1}{\pi}\int_0^{2\pi} \cos(mx)\cos(nx) = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}.$$