Wkst #14

1. (a) $\int u(x)\,v'(x)\,dx = u(x)\,v(x) - \int u'(x)\,v(x)\,dx$

(b) $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x)\,dx$

$\int_a^b \frac{d}{dx}(f(x)g(x)) = \int_a^b f(x)g'(x) + g(x)f'(x)\,dx$

$f(x)g(x)\Big|_a^b = \int_a^b f(x)g'(x)\,dx + \int_a^b g(x)f'(x)\,dx$

$\int_a^b f(x)g'(x)\,dx = f(x)g(x)\Big|_a^b - \int_a^b g(x)f'(x)\,dx$

2. g) $\int \sin^3(x)\cos^2(x)\,dx$

$\sin^3(x)\cos^2(x)\,dx = \sin^2(x)\cos^2(x)\sin(x)\,dx$

$= (1-\cos^2(x))(\cos^2(x))\sin(x)\,dx$

$u = \cos x \quad du = -\sin x\,dx$

$-\int (1-u^2)(u^2)\,du = -\int u^2 - u^4\,du$

$= -\frac{1}{3}u^3 + \frac{1}{5}u^5 + C$

$= -\frac{1}{3}\cos^3(x) + \frac{1}{5}\cos^5(x) + C$

h) $\int x^2 \sin x\,dx$ $\quad u = x^2 \quad dv = \sin x$

$du = x \quad v = -\cos x$

$x^2(-\cos x) + \int \cos x \cdot x\,dx$

$u = x \quad dv = \cos x$

$du = 1 \quad v = \sin x$

$= -x^2\cos x + x\sin x - \int \sin x\,dx$

$-x^2\cos x + x\sin x + \cos x$

(i) $\int \frac{\sin\theta}{\cos^3\theta}\,d\theta = \int \sec^2\theta \cdot \tan(\theta)\,d\theta$

$u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

$\int u\,du$

$\frac{1}{2}u^2 \Rightarrow \frac{1}{2}(\sec^2\theta) + C$

(Equivalently, $\frac{1}{2}\tan^2\theta + K$ if using $u = \tan\theta$.)

(j) $\int \tan^5(x)\sec^3(x)\,dx$

Use identity $\tan^2 x = \sec^2 x - 1$ to write

$\tan^{2k+1} x \sec^n x$ as

$(\sec^2 x - 1)^2(\sec^{3-1} x)(\sec x \tan x)$

$= (\sec^2 x - 1)^2(\sec^2 x)(\sec x \tan x)$

$u = \sec x$

$du = \sec x \tan x\,dx$

$\int (u^2 - 1)^2 \cdot u^2\,du$

$\int (u^4 - 2u^2 + 1)(u^2)\,du$

$= \int u^6 - 2u^4 + u^2\,du$

$\frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3$

$= \frac{1}{7}(\sec^7 x) - \frac{2}{5}\sec^5 x + \frac{1}{3}(\sec^3 x)$

MA 114 Worksheet 14 - Integration by Parts and Trigonometry

2) a) $\int \cos^2(x)dx = \frac{x}{2} + \frac{\sin 2x}{4} + C = \frac{x}{2} + \frac{1}{2}\sin x \cos x + C$

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b) $\int \ln t \, dt$ $\quad u = \ln t \quad dv = dt$
$\quad du = \frac{1}{t}dt \quad v = t$

$\int \ln t \, dt = uv - \int v\,du = t\ln t - \int t \cdot \frac{1}{t}dt = t\ln t - \int dt$

$= t\ln t - t + C$

c) $\int_1^2 y^4 \ln y \, dy$ $\quad u = \ln y \quad dv = y^4 dy$
$\quad du = \frac{1}{y}dy \quad v = \frac{y^5}{5}$

$$\int_1^2 y^4 \ln y\,dy = \frac{y^5}{5}\ln y \Big|_1^2 - \int_1^2 \frac{y^5}{5}\cdot\frac{1}{y}\,dy$$

$$= \frac{y^5}{5}\ln y \Big|_1^2 - \int_1^2 \frac{y^4}{5}\,dy$$

$$= \frac{y^5}{5}\ln y \Big|_1^2 - \frac{y^5}{25}\Big|_1^2$$

$$= \left[\frac{2^5}{5}\ln(2) - \cancelto{0}{\frac{1^5}{5}\ln(1)}\right] - \left[\frac{2^5}{25} - \frac{1^5}{25}\right]$$

$$= \frac{32}{5}\ln(2) - \frac{32}{25} + \frac{1}{25}$$

$$= \frac{32}{5}\ln(2) - \frac{31}{25}$$

d) $\int e^x \cos(x)dx$ $u = \cos(x)$ $dV = e^x dx$
$du = -\sin(x)dx$ $V = e^x$

$$\int e^x \cos(x)dx = e^x\cos(x) - \int -e^x\sin(x)dx + C$$
$$= e^x\cos(x) + \int e^x \sin(x)dx$$ $u = \sin(x)$ $dv = e^x dx$
$du = \cos(x)dx$ $v = e^x$
$$= e^x\cos(x) + e^x\sin(x) - \int e^x\cos(x)dx + C$$

So we have
$$\int e^x\cos(x)dx = e^x\cos(x) + e^x\sin(x) - \int e^x\cos(x)dx + C$$
add $\int e^x\cos(x)dx$ to both sides

$$2\int e^x\cos(x)dx = e^x\cos(x) + e^x\sin(x) + C$$

$$\int e^x\cos(x)dx = \frac{1}{2}\left[e^x\cos(x) + e^x\sin(x)\right] + C$$
$$= \frac{e^x}{2}\left[\cos(x) + \sin(x)\right] + C$$

e) $\int_0^{\pi/2} \sin^2(x)\cos^2(x)\,dx$ $\sin^2(x) = (1-\cos^2(x))$

$$= \int_0^{\pi/2} (1-\cos^2(x))\cos^2(x)\,dx = \int_0^{\pi/2} \cos^2(x) - \cos^4(x)\,dx$$
$$= \int_0^{\pi/2} \cos^2(x)dx - \int_0^{\pi/2}\cos^4(x)dx$$

use reduction formula on $\int_0^{\pi/2} \cos^4(x)dx$

$$\int_0^{\pi/2} \cos^4(x)dx = \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\int \cos^2 x\, dx$$

So $\int_0^{\pi/2} \cos^2(x)dx - \int_0^{\pi/2} \cos^4(x)dx$

$$= \int_0^{\pi/2} \cos^2(x)dx - \left[\frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\int \cos^2 x\, dx\right]$$

$$= -\frac{1}{4}\cos^3 x \sin x + \frac{1}{4}\int_0^{\pi/2} \cos^2(x)\, dx$$

$$= -\frac{1}{4}\cos^3 x \sin x + \frac{1}{4}\left[\frac{x}{2} + \frac{1}{2}\sin x \cos x\right]\Bigg|_0^{\pi/2}$$

$$= -\frac{1}{4}\cos^3 x \sin x + \frac{1}{4}\left[\frac{\pi/2}{2} + \frac{1}{2}\sin(\pi/2)\cos(\pi/2)\right] - \frac{1}{4}\left[\frac{0}{2} + \frac{1}{2}\sin(0)\cos(0)\right]$$

$$= -\frac{1}{4}\cos^3 x \sin x + \frac{1}{4}\left[\frac{\pi}{4} + \frac{1}{2}(1)(0)\right] - \frac{1}{4}[0]$$

$$= -\frac{1}{4}\cos^3 x \sin x + \frac{\pi}{8}$$

$$= \frac{\pi}{8} - \frac{1}{4}\cos^3 x \sin x$$

f) $\int \cos(\ln x)\,dx$ $\quad u = \cos(\ln x) \quad dv = dx$

$du = -\frac{\sin(\ln x)}{x}dx \quad v = x$

$$\int \cos(\ln x)dx = x\cos(\ln x) - \int \frac{-x\sin(\ln x)dx}{x} = x\cos(\ln x) + \int \sin(\ln x)dx$$

$u = \sin(\ln x) \quad dv = dx$

$du = \frac{\cos(\ln x)}{x} \quad v = x$

$$= x\cos(\ln x) + x\sin(\ln x) - \int \frac{x\cos(\ln x)}{x}dx$$

$$= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)dx$$

$$\int \cos(\ln x)dx = x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)dx$$

add $\int \cos(\ln x)dx$ to both sides

$$2\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x)$$

$$\int \cos(\ln x)\,dx = \frac{1}{2}\left[x\cos(\ln x) + x\sin(\ln x)\right]$$

$$\int \cos(\ln x)dx = \frac{x}{2}\left[\cos(\ln x) + \sin(\ln x)\right]$$

3. If $m = \pm n$, we have $\frac{1}{\pi}\int_0^{2\pi} \cos^2(nx)\,dx$.
Put $u = nx$. Then $du = n\,dx$. So the integral becomes

$$\frac{1}{n\pi}\int_0^{2\pi n} \cos^2(u)\,du$$

$$= \frac{1}{n\pi}\left[\frac{u}{2} + \frac{\sin(2u)}{4}\right]_0^{2\pi n}$$

$$= \frac{1}{n\pi}\left[\pi n + \frac{\sin(4\pi n)}{4}\right] = \frac{\pi n}{n\pi} = 1.$$

If $m \neq \pm n$, the integral becomes

$$\frac{1}{\pi}\int_0^{2\pi} \cos(mx)\cos(nx)\,dx = \frac{1}{\pi}\left[\frac{\sin((m-n)x)}{2(m-n)} + \frac{\sin((m+n)x)}{2(m+n)}\right]_0^{2\pi}$$

$$= 0.$$