

MA 114 Worksheet # 15: Integration by Trigonometric Substitution

1. Conceptual Understanding:

(a) Given the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, prove that:

$$\sec^2(\theta) = \tan^2(\theta) + 1.$$

(b) Given $x = a \sin(\theta)$ with $a > 0$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, show that $\sqrt{a^2 - x^2} = a \cos(\theta)$.

(c) Given $x = a \tan(\theta)$ with $a > 0$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, show that $\sqrt{a^2 + x^2} = a \sec(\theta)$

(d) Given $x = a \sec(\theta)$ with $a > 0$ and $0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$, show that $\sqrt{x^2 - a^2} = a \tan(\theta)$.

2. Compute the following integrals:

(a) $\int_0^2 \frac{u^3}{\sqrt{16 - u^2}} du$

(b) $\int \frac{1}{x^2 \sqrt{25 - x^2}} dx$

(c) $\int \sqrt{5 + 4x - x^2} dx$

(d) $\int \frac{x^3}{\sqrt{64 + x^2}} dx$

(e) $\int_0^1 \sqrt{x^2 + 1} dx$

(f) $\int \frac{x}{\sqrt{x^2 + 1}} dx$

3. Let $a, b > 0$. Prove that the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab .