

MA 114 Worksheet # 16: Exam II Review

1. Taylor Series: Find the Taylor expansions centered at $x = a$ for each of the following functions. Give the associated radius of convergence for each series.

(a) $f(x) = x - x^3$, $a = -2$.

(b) $f(x) = \frac{1}{x}$, $a = 1$

(c) $f(x) = e^{-x^2} + e^{x^2}$, $a = 0$

(d) $f(x) = \ln(1 + x^2)$, $a = 0$

2. A spring has natural length of 20 cm. Compare the work W_1 done in stretching the spring from 20 cm to 30 cm with the work W_2 done in stretching it from 30 cm to 40 cm. How are W_1 and W_2 related?

3. Areas and Volumes

(a) Find the volume of the solid obtained by rotating the region bounded by the curves $\sqrt{25 - x^2}$, $y = 0$ about the x -axis.

(b) Find the area of the region enclosed by the curves $y = 3x^2$, $y = 8x^2$, $4x + y = 4$ for $x \geq 0$. Leave your answer in terms of integrals. [Hint: one root of $8x^2 + 4x - 4$ is $x = 1/2$ and one root of $3x^2 + 4x - 4$ is $x = 2/3$.]

(c) Find the area of the region enclosed by the curves $x = |y|$ and $x = y^2 - 2$.

(d) Find the volume of the solid obtained by rotating the region bounded by the curves $y = 1/x$, $y = 0$, $x = 1$, $x = 3$ about the line $y = -1$. Leave your answer in terms of integrals.

(e) Use cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$, $y = 2 - x^2$ about the line $x = 5$. Leave your answer in terms of integrals.

(f) Find the volume of the solid obtained by rotating the region bounded by the curves $x + y = 3$, $x = 4 - (y - 1)^2$ about the x -axis. Leave your answer in terms of integrals.

4. Integration: Compute the following integrals.

(a) $\int t \sin(t) dt$

(b) $\int \ln x dx$

(c) $\int \sin(t) \cot(t) dt$

(d) $\int \sin^2(t) \cos^3(t) dt$

(e) $\int \frac{t}{\sqrt{4 - t^2}} dt$

(f) $\int \tan^2(t) dt$

(g) $\int \frac{1}{\sqrt{4 + t^2}} dt$

(h) $\int \tan^3(\theta) \sec^3(\theta) d\theta$

5. A cup of coffee, which is initially 200° F, cools in a 75° F room. The function

$$T(t) = 75 + 125e^{-.02t}$$

gives the temperature of the coffee at t minutes. Find the average temperature of the coffee during the second half-hour of cooling.