MA 114 Worksheet # 18: Improper Integrals and Integral Test

1. For each of the following, determine if the integral is proper or improper. If it is improper, explain why. Do not evaluate any of the integrals.

   (a) $\displaystyle\int_0^2 \frac{x}{x^2-5x+6}\,dx$

   (b) $\displaystyle\int_1^2 \frac{1}{2x-1}\,dx$

   (c) $\displaystyle\int_1^2 \ln(x-1)\,dx$

   (d) $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{1+x^2}\,dx$

   (e) $\displaystyle\int_0^{\pi/2} \sec x\,dx$

2. For the integrals below, determine if the integral is convergent or divergent. Evaluate the convergent integrals.

   (a) $\displaystyle\int_{-\infty}^{0} \frac{1}{2x-1}\,dx$

   (b) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\,dx$

   (c) $\displaystyle\int_0^{\infty} \frac{1}{x}\,dx$

   (d) $\displaystyle\int_{10^{-10}}^{\infty} \frac{1}{x^2}\,dx$

   (e) $\displaystyle\int_0^1 \ln(x)\,dx$

   (f) $\displaystyle\int_0^{\infty} \sin\theta\,d\theta$

3. Consider the integral

$$\int_1^{\infty} \frac{1}{x^p}\,dx.$$

   For what values of p does this integral converge? For what values of p does the integral diverge?

4. Use the integral test to prove that a p-series converges for $p > 1$ and diverges for $p \leq 1$.

5. Use the integral test to determine if the following series converge or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n^{3/2}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+5}$