MA 114 Worksheet # 19: Arc Length

1. Conceptual Understanding:

   (a) Let $y = f(x)$ be a continuous curve and $[a, b]$ an interval. Let $a = x_0 < x_1 < x_2 < \ldots < x_n = b$ divide the interval into n even subintervals. Write down a summation which approximates the length of the curve.

   (b) Explain how to obtain the arc-length formula by taking a limit in part (a).

   (c) Use calculus to establish the formula for the circumference of a circle.

   (d) Write down an integral which gives the length of the sine curve over one period. Use a calculator to find an approximation for this value.

2. Find the arc length of the curve over the given interval.

   (a) $y = \ln(\sec x); \qquad 0 \leq x \leq \pi/4$

   (b) $y = \dfrac{x^2}{2}; \qquad [0, 1]$

3. (Text, problem 21, p 530.) Use your calculator to plot the curve

$$y = \frac{2}{3}(x^2 - 1)^{3/2}$$

   over the interval from 1 to 3. Use the graph to estimate the length of the curve. Compute the exact length and compare to your approximation.

4. (Text, problem 38, p 531.) Recall from Worksheet 17 that the Gateway Arch is modeled by the equation

$$y = 211.49 - 20.96 \cosh(.03291765x).$$

   where $|x| \leq 91.20$ and the dimensions are in meters. Give an integral formula for the length of the arch and then approximate the value of the integral with your calculator.

5. Computing the length of curves is an important application of calculus. Unfortunately, it is not usually possible to obtain an exact numerical expression for arc length because the integral in the arc length formula often has no closed form expression. In such situations arc length can be approximated by using an appropriate numerical approximation for the integral. In the following write down an integral expression for the length of the curve but do not evaluate the integral.

   (a) $y = e^{x^2}$ on $[0, 1]$

   (b) $y = \tan x$ on $[0, \pi/4]$

   (c) $x = y + y^3$ for $1 \leq y \leq 4$

   (d) $y = \arctan x$ on $[0, 1]$

   (e) The ellipse $\dfrac{y^2}{a^2} + \dfrac{x^2}{b^2} = 1$