Worksheet 1 Key

1. a) A <u>sequence</u> is an ordered collection of numbers $a_1, a_2, a_3, \ldots$

b) $\lim_{x\to\infty} f(x) = L$ means that as x approaches ∞, $f(x)$ approaches L. We usually think of x as a real number. In contrast, $\lim_{n\to\infty} f(n) = L$ means that for any increasing sequence $a_1, a_2, \ldots$ the sequence $f(a_1), f(a_2), f(a_3), \ldots$ approaches L. (x is continuous, n is discrete.)

c) A sequence converges if the terms of the sequence "settle in" on a particular value. Formally, a sequence $\{a_n\}$ converges to L if for all $\varepsilon > 0$ there exists some N so that whenever $n > N$, we have $|a_n - L| < \varepsilon$. That is, $\lim_{n\to\infty} a_n = L$.

d) A sequence $\{a_n\}$ diverges if $\lim_{n\to\infty} a_n$ is $\pm\infty$, or does not exist.

e) $\{\frac{1}{n}\} \to 0$ $\quad \{(-1)^n \frac{1}{n^2}\} \to 0$

$\{1, 2, 3, 3, 3, 3, \ldots\} \to 3$

$\left\{\left(1 + \frac{1}{n}\right)^n\right\} \to e$ (Answers may vary.)

$\{3, 3.1, 3.14, 3.141, 3.1415, \ldots\} \to \pi$

2. a) $\{3, \frac{3}{2}, \frac{3}{3}, \frac{3}{4}, \ldots\} \to 0$

b) $\{\frac{1}{2}+2, \frac{1}{4}+2, \frac{1}{8}+2, \frac{1}{16}+2, \ldots\} \to 2$

c) $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\} \to 1$

d) $\{-\frac{1}{1}, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, \ldots\} \to 0$

e) $\{1, 1, 2, 3, 5, 8, \ldots\}$ does not converge. This is the Fibonacci sequence.

3. a) $\{\frac{1}{n}\}_{n=1}^{\infty}$

b) $a_1 = 0$, $a_n = 1 - a_{n-1}$ for each $n > 1$.

c) $a_n = \frac{-n}{2^{\frac{n+1}{2}}}$ if n odd, $\quad a_n = \frac{n}{3^{n/2}}$ if n even

4. a) Yes, using the limit laws for sequences with multiplication, division, and addition, the limit is $\frac{15 \cdot 1}{0+1} = 15$.

b) Again, using the limit laws we see that the denominator approaches zero, while the numerator approaches 18. Thus the sequence diverges to positive infinity.

5. a) $$\lim_{n\to\infty} \frac{n^2+n+1}{3n^2+2n+15} \overset{Thm}{=} \lim_{x\to\infty} \frac{x^2+x+1}{3x^2+2x+15}$$
$$\overset{LH}{=} \lim_{x\to\infty} \frac{2x+1}{6x+2} \overset{LH}{=} \frac{2}{6} = \frac{1}{3}$$

b) $$\lim_{n\to\infty} \frac{n^2}{e^n} \overset{Thm}{=} \lim_{x\to\infty} \frac{x^2}{e^x} \overset{LH}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{LH}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0$$

c) $$\lim_{n\to\infty} \frac{(\ln(n))^2}{n} \overset{Thm}{=} \lim_{x\to\infty} \frac{(\ln(x))^2}{x}$$
$$\overset{LH}{=} \lim_{x\to\infty} \frac{2\ln(x)}{x}$$
$$\overset{LH}{=} \lim_{x\to\infty} \frac{2}{x} = 0$$