# MA 114 Worksheet # 2: Convergence of Sequences and Series

1. Conceptual Understanding:

   (a) What is a partial sum of a sequence? Give the definition for convergence of a series.

   (b) Explain the difference between a sequence and a series.

   (c) Give an example of a bounded sequence that does not converge.

   (d) Suppose the sequence $\{a_n\}$ is bounded above by $M$ and increases monotonically. Does it follow that the sequence must converge to $M$?

   (e) Some textbooks call the series

   $$\sum_{n=1}^{\infty} ar^{n-1}$$

   a *geometric series* when $a$ and $r$ are nonzero constants. Is this the same definition as in Rogawski? For what values of $r$ does this series converge? For what values of $r$ does this series diverge? What is the the sum if the series converges?

2. Review: Working with inequalities. Inequalities will be an important tool for Calculus II. In this section, you will manipulate inequalities to prove that sequences are monotonic or bounded.

   (a) Recall the following rules for inequalities

   $$(1) \qquad \text{If } a \le c \text{ and } b > 0 \text{ then } \frac{a}{b} \le \frac{c}{b}$$

   $$(2) \qquad \text{If } 0 < c \le b \text{ and } a > 0 \text{ then } \frac{a}{b} \le \frac{a}{c}$$

   (b) Use the rules to prove each of the inequalities:

   $$\frac{1}{n+1} \le \frac{1}{n}, \quad \frac{2n^2}{2n+1} \le n, \quad \frac{2n-1}{3n+2} \le \frac{2}{3}, \quad \frac{n+1}{2n^3+4} \le \frac{1}{n^2}.$$

3. Explain how the first inequality in 2(b) proves that the sequence $\{\frac{1}{n}\}$ is monotonic. Find a bound for the sequence and argue that the sequence converges.

4. (a) Explain how you could use the first derivative test to show that a sequence is monotonically increasing.

   (b) Let $a_n = \dfrac{n}{n^2+1}$ and consider the sequence $\{a_n\}_{n=1}^{\infty}$. For what continuous, infinitely differentiable function $f$ does $f(n) = a_n$? Argue that this function is a monotonic function and use this to prove that the sequence is also monotonic.

   (c) Proceed to prove that the sequence $\{a_n\}_{n=1}^{\infty}$ is convergent.

5. First check if the following sequences are bounded and/or monotonic. Then decide whether each sequence converges or diverges.

   (a) $\{1 + \frac{1}{n}\}$

   (b) $\{(-1)^n/n\}$

   (c) $\{a_n\}$ where $a_n = \dfrac{n}{e^n}$

6. Do the following series converge or diverge? For each convergent series find the sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n$

   (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-3}{2}\right)^{n-1}$

   (c) $5 - \dfrac{5}{4} + \dfrac{5}{4^2} - \dfrac{5}{4^3} + \cdots$

7. Rewrite the repeated decimal $0.\overline{04}$ as an infinite series. Use the series representation to write the number as a fraction.