MA 114 Worksheet # 21: Parametric Curves

1. Conceptual Understanding:

   (a) What parametric equations represent the circle of radius 5 with center $(2, -4)$?

   (b) Represent the ellipse
   $$\frac{y^2}{b^2} + \frac{x^2}{a^2} = 1$$
   with parametric equations.

   (c) True or false? The two sets of parametric equations
   $$y_1(t) = 5\sin(t), \quad x_1(t) = 5\cos(t), \quad 0 \leq t \leq 2\pi$$
   and
   $$y_2(t) = 5\sin(t), \quad x_1(t) = 5\cos(t), \quad 0 \leq t \leq 20\pi$$
   represent the same parametric curve. Discuss.

2. Sketch the curve represented by the following parametric equations. Be sure to indicate direction.

   (a) $x = 3\cos(t)$, $y = -2\sin(t)$, $0 \leq t \leq 2\pi$.

   (b) $x = t^2 - 1$, $y = t^2 + 1$, $-2 \leq t \leq 2$.

   (c) $x = \frac{1}{t}$, $y = t + 1$, $\frac{1}{2} \leq t \leq 5$.

3. Find a Cartesian equation for the following parametric curves.

   (a) $x = \sinh(t), \quad y = \cosh(t)$

   (b) $x = \sqrt{t}, \quad y = 1 - t$

   (c) $x = t^2, \quad y = t^4$

4. Represent each of the following curves as parametric equations traced just once on the indicated interval.

   (a) $y = x^3$ from $x = 0$ to $x = 2$

   (b) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

5. A particle travels from the point $(2, 3)$ to $(-1, -1)$ along a straight line over the course of 5 seconds. Write down a set of parametric equations which describe the position of the particle for any time between 0 and 5 seconds. If the particle returns from $(2, 3)$ to $(-1, -1)$ in the same amount of time, what parametric equations describe its journey back?