Worksheet 21 Key

1. a) Cartesian equation $(x-2)^2 + (y+4)^2 = 5^2$

Put $x - 2 = 5\cos(t)$, $y + 4 = 5\sin(t)$.

$x = 5\cos(t) + 2$ $\quad$ $y = 5\sin(t) - 4$

One should check that these satisfy the Cartesian equation.

b) Cartesian equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Put $x = a\cos(t)$, $y = b\sin(t)$.

c) This is true in the sense that both have the same graph in the xy plane, of a circle of radius 5. But for $0 \le t \le 2\pi$, the circle is traced out exactly once counterclockwise. For $0 \le t \le 20\pi$, the circle is traced out ten times.

3. a) $x = \sinh(t) = \frac{1}{2}(e^t - e^{-t}) \Rightarrow 2x = e^t - e^{-t}$

$y = \cosh(t) = \frac{1}{2}(e^t + e^{-t}) \Rightarrow 2y = e^t + e^{-t}$

Add down: $2(x+y) = 2e^t$

$x + y = e^t$

$t = \ln(x+y)$

Sub back in: $2x = e^{\ln(x+y)} - e^{-\ln(x+y)}$

$2x = (x+y) - \frac{1}{x+y}$

b) $x = \sqrt{t}$

$y = 1 - t \Rightarrow t = 1 - y$

Substitute: $x = \sqrt{1-y}$

c) $x = t^2$

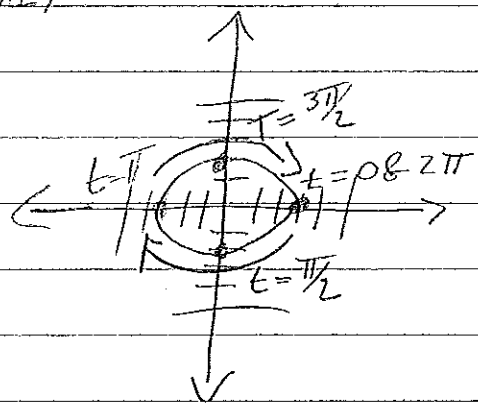
$y = t^4 = (t^2)^2$

So $y = x^2$

Sketch curve represented by following parametric eqs.

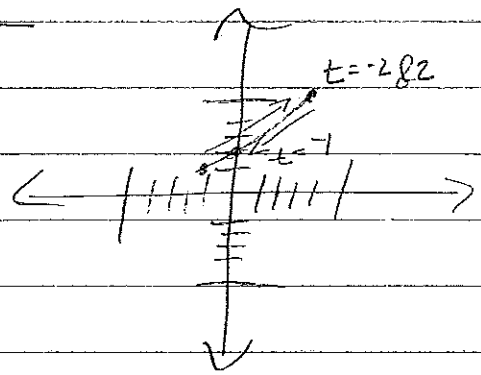
2a)

t	$x=3\cos(t)$	$y=-2\sin(t)$
0	3	0
$\pi/2$	0	-2
π	-3	0
$3\pi/2$	0	2
2π	3	0



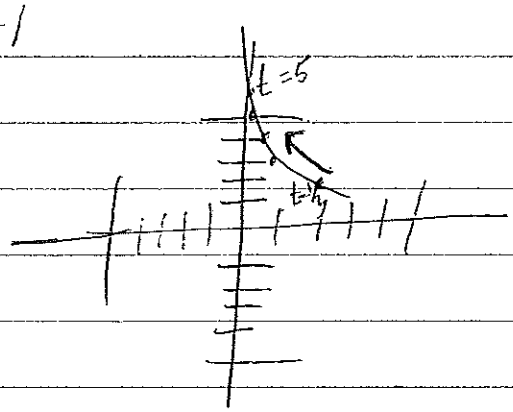
2b)

t	$x=t^2-1$	$y=t^2+1$
-2	3	5
-1	0	2
0	-1	1
1	0	2
2	3	5



2c)

t	$x=\frac{1}{t}$	$y=t+1$
$\frac{1}{2}$	2	$1\frac{1}{2}$
1	1	2
2	$\frac{1}{2}$	3
4	$\frac{1}{4}$	5
5	$\frac{1}{5}$	6



4 Represent curves traced just once on indicated interval

(a) $y = x^3$

$c(t) = (t, t^3)$ from $t = 0$ to $t = 2$

b) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$c(t) = (2\cos t, 3\sin t)$ for $0 \leq t < 2\pi$

MA 114 Worksheet 21 - Parametric Curves

5)

t	x
0	2
5	-1

$$\frac{-1-2}{5-0} = \frac{-3}{5}$$

$$x = \frac{-3}{5}t + b$$

$$2 = \frac{-3}{5}(0) + b$$

$$b = 2$$

$$x = \frac{-3}{5}t + 2$$

x	y
2	3
-1	-1

$$\frac{3+1}{2+1} = \frac{4}{3}$$

$$y = \frac{4}{3}x + b$$

$$-1 = \frac{4}{3}(-1) + b$$

$$-1 = -\frac{4}{3} + b$$

$$b = \frac{1}{3}$$

$$y = \frac{4}{3}x + \frac{1}{3}$$

$$y = \frac{4}{3}\left(\frac{-3}{5}t + 2\right) + \frac{1}{3}$$

$$y = -\frac{4}{5}t + \frac{8}{3} + \frac{1}{3}$$

$$y = -\frac{4}{5}t + 3$$

Original Trip: $x = \frac{-3}{5}t + 2$

$y = -\frac{4}{5}t + 3$

Return Trip: Replace t with $5-t$

$$x = -\frac{3}{5}(5-t) + 2 = -3 + \frac{3}{5}t + 2 = \frac{3}{5}t - 1$$

$$y = -\frac{4}{5}(5-t) + 3 = -4 + \frac{4}{5}t + 3 = \frac{4}{5}t - 1$$