MA 114 Worksheet # 22: Calculus with Parametric Curves

1. For the following parametric curves, find an equation for the tangent to the curve at the specified value of the parameter.

   (a) $x = e^{\sqrt{t}}$, $y = t - \ln t^2$ at $t = 1$.

   (b) $x = \cos(\theta) + \sin(2\theta)$, $y = \cos(\theta)$, at $\theta = \pi/2$.

2. For the following parametric curve, find dy/dx.

   (a) $x = t - e^t$, $y = t + e^{-t}$.

   (b) $x = t^3 - 12t$, $y = t^2 - t$.

   (c) $x = 2\cos(2t)$, $y = \cos(t)$, $0 < t < 2\pi$

3. Find d^2y/dx^2 for the curve $x = 7 + t^2 + e^t$, $y = \cos(t) + \frac{1}{t}$, $0 < t \leq \pi$.

4. Find an example which shows that
$$\frac{d^2y}{dx^2} \neq \frac{\dfrac{d^2y}{dt^2}}{\dfrac{d^2x}{dt^2}}.$$

5. Use parametric equations to find a formula for the area of the ellipse:
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

   [Note: The area formula in 10.2 assumes the direction in which the parametric curve is traced out is clockwise. For curves traced out in a counterclockwise direction, insert a negative sign into the formula.]

6. Find the length of the curve $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leq t \leq 1$.