M1:114 Worksheet 23 - Polar Coordinates

1) a) $(x,y) = (2,-2)$

$r = \sqrt{2^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{-2}{2} = -1 \quad \theta = \tan^{-1}(-1) = -\pi/4$

Polar Coordinates: $(2\sqrt{2}, -\pi/4)$

b) $(x,y) = (1,-2)$

$r = \sqrt{1^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

$\tan\theta = \frac{y}{x} = \frac{-2}{1} = -2 \quad \theta = \tan^{-1}(-2)$

Polar coordinates: $(\sqrt{5}, \tan^{-1}(-2))$

c) $(-1, \sqrt{3})$

$r = \sqrt{(-1)^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3} \quad \theta = \tan^{-1}(-\sqrt{3}) = -\pi/3 + \pi$

$= \frac{2\pi}{3}$ (second quadrant)

Polar coordinates: $(2, 2\pi/3)$

5) (r_1, θ_1) gives $x_1 = r_1\cos\theta_1$, $y_1 = r_1\sin\theta_1$ $\quad (r_1\cos\theta_1, r_1\sin\theta_1)$

(r_2, θ_2) gives $x_2 = r_2\cos\theta_2$, $y_2 = r_2\sin\theta_2$ $\quad (r_2\cos\theta_2, r_2\sin\theta_2)$

$d = \sqrt{(r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1 - r_2\sin\theta_2)^2}$

$= \sqrt{r_1^2\cos^2\theta_1 - 2r_1r_2\cos\theta_1\cos\theta_2 + r_2^2\cos^2\theta_2 + r_1^2\sin^2\theta_1 - 2r_1r_2\sin\theta_1\sin\theta_2 + r_2^2\sin^2\theta_2}$

$= \sqrt{r_1^2(\cos^2\theta_1 + \sin^2\theta_1) + r_2^2(\cos^2\theta_2 + \sin^2\theta_2) - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)}$

$$= \sqrt{r_1^2 + r_2^2 - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)}$$

(r, θ)

2 a $(1, \pi) \rightarrow (1\cos(\pi), 1\sin(\pi))$
$= (-1, 0)$

b) $(-1, \pi/2) \rightarrow (-1\cos(\frac{\pi}{2}), -1\sin(\frac{\pi}{2}))$
$= (0, -1)$

c) $(1, -\pi/4) \rightarrow (1\cos(-\pi/4), 1\sin(-\pi/4))$
$= (\frac{\sqrt{2}}{2}, -\sqrt{2}/2)$

4a) $r = 3$

b) $y + x = 4$
$r\sin\theta + r\cos\theta = 4 \Rightarrow r = \dfrac{4}{\sin\theta + \cos\theta}$

c) $x = 4$
Choose $(4, 0)$ to be (d, α)
$r = 4\sec(\theta)$

d) $xy = 4$
$p_0 = (2,2)$ or $(-2,-2)$
~~$r = 2\sec(\theta - 2)$~~
$xy = 4$
$r^2 \sin\theta\cos\theta = 4$
$r = \sqrt{\dfrac{4}{\sin\theta\cos\theta}}$

a) $r = 2\cos\theta + 1$

$x = 2\cos^2\theta + \cos\theta \qquad y = 2\cos\theta\sin\theta + \sin\theta$

$x' = 4\cos\theta \cdot -\sin\theta - \sin\theta \qquad y' = 2\cos\theta\cos\theta - 2\sin\theta\sin\theta + \cos\theta$

$= -4\sin\theta\cos\theta - \sin\theta \qquad = 2\cos^2\theta - 2\sin^2\theta + \cos\theta$

$$\frac{dy}{dx} = \frac{2\cos^2\theta - 2\sin^2\theta + \cos\theta}{-4\sin\theta\cos\theta - \sin\theta}$$

b) $r = 1/\theta$

$x = \frac{1}{\theta}\cos\theta \qquad y = \frac{1}{\theta}\sin\theta$

$x' = \frac{-\theta\sin\theta - \cos\theta(1)}{\theta^2} \qquad y' = \frac{\theta\cos\theta - \sin\theta(1)}{\theta^2}$

$= \frac{-\theta\sin\theta - \cos\theta}{\theta^2} \qquad = \frac{\theta\cos\theta - \sin\theta}{\theta^2}$

$$\frac{dy}{dx} = \frac{\theta\cos\theta - \sin\theta}{\theta^2} \cdot \frac{\theta^2}{-\theta\sin\theta - \cos\theta}$$

$$= \frac{\theta\cos\theta - \sin\theta}{-\theta\sin\theta - \cos\theta}$$

c) $r = 2e^{-\theta}$

$x = 2e^{-\theta}\cdot\cos\theta \qquad y = 2e^{-\theta}\sin\theta$

$x' = 2e^{-\theta}\cdot -\sin\theta - 2e^{-\theta}\cos\theta \qquad y' = 2e^{-\theta}\cos\theta - 2e^{-\theta}\sin\theta$

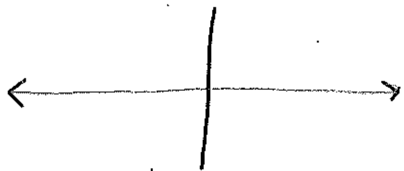
$= -2e^{-\theta}\sin\theta - 2e^{-\theta}\cos\theta \qquad = 2e^{-\theta}(\cos\theta - \sin\theta)$

$= -2e^{-\theta}(\sin\theta + \cos\theta)$

$$\frac{dy}{dx} = \frac{2e^{-\theta}(\cos\theta - \sin\theta)}{-2e^{-\theta}(\sin\theta + \cos\theta)}$$

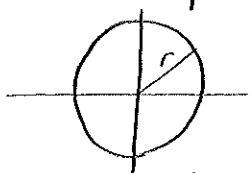
$$= \frac{\cos\theta - \sin\theta}{-\sin\theta - \cos\theta}$$

3. a) $\theta = -\pi$



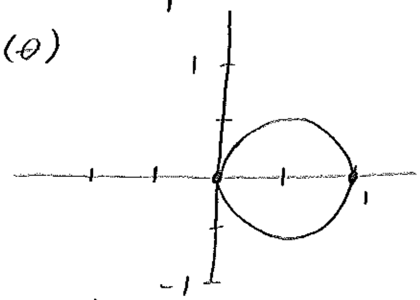
line $y=0$

b) $r = -\pi$



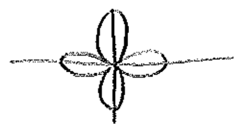
circle radius π

c) $r = \cos(\theta)$



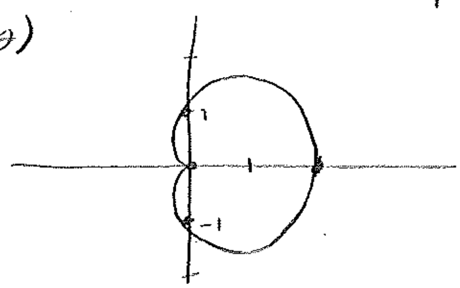
θ	r
0	1
$\pi/2$	0
π	-1

d) $r = \cos(2\theta)$



Make θ, r chart.

e) $r = 1 + \cos(\theta)$



θ	r
0	2
$\pi/2$	1
π	0
$3\pi/2$	1

7. a) $r = \sin(\theta)$. $x = \sin(\theta)\cos(\theta)$ $y = \sin^2(\theta)$

Then $x'(\theta) = -\sin^2(\theta) + \cos^2(\theta)$

$y'(\theta) = 2\sin(\theta)\cos(\theta)$

$$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} = \frac{2\sin(\theta)\cos(\theta)}{\cos^2(\theta) - \sin^2(\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\sqrt{3}/2}{\frac{1}{4} - \frac{3}{4}} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}.$$

b) $r = 1/\theta$, $x = 1/\theta \cos(\theta)$ $y = 1/\theta \sin(\theta)$

$x'(\theta) = -1/\theta^2 \cos(\theta) - 1/\theta \sin(\theta)$

$y'(\theta) = -1/\theta^2 \sin(\theta) + 1/\theta \cos(\theta)$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{-\left(\frac{2}{\pi}\right)^2}{-\frac{2}{\pi}} = \frac{2}{\pi}$$