MA 114 Worksheet # 23: Review for Exam III

1. Partial fractions: Find the partial fraction decomposition for each of the following but do not determine the coefficients.

   (a) $\dfrac{x^2+1}{x^2+3x+2}$

   (b) $\dfrac{x+2}{(x^2+1)^2(x+5)(x+3)}$

   (c) Compute
   $$\int \frac{x+2}{(x+5)(x-2)}\,dx.$$

2. Improper integrals: Determine if the following integrals converge or diverge. Determine the values of the convergent integrals.

   (a) $\displaystyle\int_0^\infty \frac{x}{(x^2+1)^2}\,dx$

   (b) $\displaystyle\int_{-\infty}^\infty \frac{x}{x^2+1}\,dx$

   (c) $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

   (d) $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^3}\,dx$

3. Use the comparison theorem for improper integrals to establish the convergence or divergence of the integral
   $$\int_1^\infty \frac{2+\sin x}{x^3+x}\,dx.$$

4. Use the integral test to establish the convergence or divergence of the series
   $$\sum_{n=1}^\infty \frac{1}{n^2+1}.$$

   What is the largest possible error we could obtain if we approximate the sum of this series with its 10th partial sum? What is the smallest possible error using this approximation?

5. Arc length and surface area:

   (a) Find the length of the arc $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ over the interval $[1, 2e]$.

   (b) Find the area of the surface of revolution when the arc $y = \sin(x)$ over the interval $[0, \pi]$ is rotated about the x-axis.

6. Parametric equations:

   (a) The motion of a particle in the plane obeys the parametric equations $x(t) = 2\cos t + 5$ and $y(t) = 3\sin t + 1$ for $0 \le t \le \pi$. Sketch the path of the particle and indicate the direction of travel.

   (b) Compute dy/dx and d^2y/dx^2 for the parametrically defined curve $x(t) = t + e^t$ and $y(t) = e^{-t}$.

7. Polar coordinates

   (a) Graph the polar equations $r = -\sec\theta$, $r = \sin\theta$ and $r = \theta$.

   (b) Convert the equation $x^2 + (y-1)^2 = 1$ to polar coordinates.

   (c) Convert the equation $r = 2\sin\theta + 2\cos\theta$ to rectangular coordinates and graph.

   (d) Find the slope of the tangent line to the polar curve $r = \theta$ at $\pi/2$.