MA 114 Worksheet # 26: Calculus with Polar Coordinates II and Differential Equations

1. Polar Coordinates (continued)

   (a) Find the area of the region that lies inside both the curves $r = \sqrt{3}\sin\theta$ and $r = \cos\theta$.

   (b) Find the area in the first quadrant that lies inside the curve $r = 2\cos\theta$ and outside the curve $r = 1$.

   (c) Find the length of the curve $r = \theta^2$ for $0 \leq \theta \leq 2\pi$.

   (d) Write down an integral expression for the length of the curve $r = \sin\theta + \theta$ for $0 \leq \theta \leq \pi$ but do not compute the integral.

2. Conceptual Understanding for Differential Equations:

   (a) Is $y = \sin(3x) + 2e^{4x}$ a solution of the differential equation $y'' + 9y = 50e^{4x}$? Explain why or why not.

   (b) Explain why every solution of
   $$\frac{dy}{dx} = y^2 + 6$$
   must be an increasing function.

   (c) What does is mean to say that a differential equation is linear or nonlinear.

3. Find all values of r so that $y(x) = e^{rx}$ is a solution of the differential equation
   $$y'' + y' - 12y = 0.$$

4. Use separation of variables to find the general solutions to the following differential equations.

   (a) $y' + 4xy^2 = 0$

   (b) $\sqrt{1-x^2}y' = xy$

   (c) $(1+x^2)y' = x^3y$

   (d) $\sqrt{1+y^2}y' + \sec x = 0$

5. Solve $y' = 4y + 24$ subject to the condition that $y(0) = 5$.

6. Solve $y' + 6y = 12$ subject to the condition that $y(2) = 10$.