MA 114 Worksheet # 27 Differential Equation Models

1. Show that the function $f(x) = x + 3e^{-x}$ solves the differential equation $\frac{dy}{dx} + y = x + 1$.

2. Basic modeling problems:

   (a) A very simple model of a human population assumes that the population grows at a rate proportional to the current population level. Based on this assumption, suggest a differential equation whose solution is the human population at time t. Discuss some of the weaknesses of this model.

   (b) Assume that a spherical raindrop evaporates at rate proportional to its surface area. Suggest a differential equation which describes how the volume of the raindrop changes over time. [Hint: Write the surface area of the sphere in terms of the volume of the sphere.] Does the assumption about the raindrop's evaporation make physical sense? Explain.

   (c) For a large object in free-fall near the earth's surface it is sometimes reasonable to assume that the resistance force due to air (drag force) is proportional to the square of the object's velocity. Suggest a differential equation whose solution is the function $v(t)$ which gives the velocity of the object at time t. [Hint: Force = mass $\cdot$ acceleration.]

3. Newton's law of cooling states that if a hot object is placed in a cool environment with constant temperture T_0 then the rate at which the temperature $y(t)$ of the object decreases is proportional to the difference between the temperatures. This means that there is some constant k so that the differential equation $y' = -k(y - T_0)$ holds.

4. Mr. Green's car runs at 212° F. On a day where the temperature is 70° F, he turns off the ignition and notes that five minutes later the engine has cooled to 160° F.

   (a) Find the cooling constant k.

   (b) When will the engine cool to 100°?

5. A cup of coffee with cooling constant $k = 0.09/\text{min}$ is placed in a room of temperature 20° C.

   (a) How quickly is the coffee cooling when the temperature is 80° C?

   (b) Use the linear approximation to estimate the change in temperature over the next 6 seconds when the temperature is 80° C.

   (c) If the coffee is initially served at 90° C, how long will it take to reach an optimal drinking temperature of 65° C?

6. A tank has the shape of the parabola $y = x^2$ revolved about the y-axis. Water leaks from a hole of area $B = 0.0005\,\text{m}^2$ at the bottom of the tank. Let $y(t)$ be the water level at time t. How long does it take for the tank to empty if the initial water level is $y(0) = 1m$?