MA 114 Worksheet # 28: Graphical Methods

1. Match the differential equation with its slope field. Give reasons for your answer.

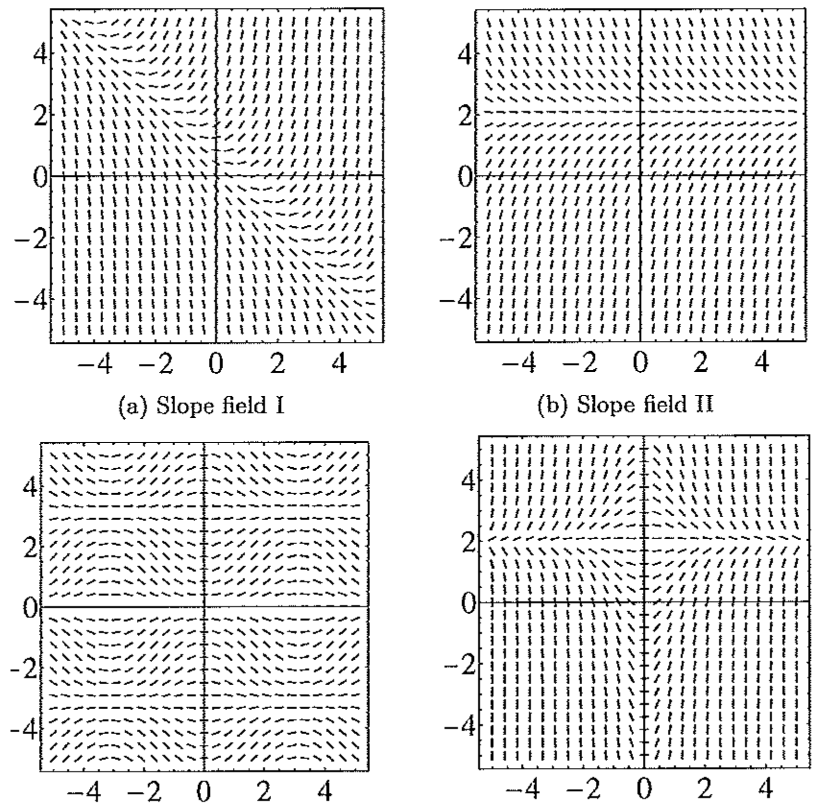
$$y' = 2 - y \qquad y' = x(2 - y) \qquad y' = x + y - 1 \qquad y' = \sin(x)\sin(y)$$

$y' = x + y - 1$
Positive above the line $x + y - 1 = 0$ and negative below.

$y' = 2 - y$
Zero when $y = 2$, does not change with x.

$y' = \sin(x)\sin(y)$
Oscillatory behavior.

$y' = x(2 - y)$
Zero when $x = 0$ or $y = 2$.



(a) Slope field I

(b) Slope field II

(c) Slope Field III

(d) Slope field IV

Figure 1: Slope fields for Problem 1

2. Use slope field labeled IV to sketch the graphs of the solutions that satisfy the given initial conditions

$$y(0) = -1, \quad y(0) = 0, \quad y(0) = 1.$$

3. Sketch the slope field of the differential equation. Then use it to sketch a solution curve that passes through the given point

   (a) $y' = y - 2x$, $(1, 0)$
   (b) $y' = xy - x^2$, $(0, 1)$

4. Show that the isoclines of $y' = t$ are vertical lines. Sketch the slope field for $-2 \le t \le 2$, $-2 \le y \le 2$ and plot the integral curves passing through $(0, 1)$ and $(0, -1)$.

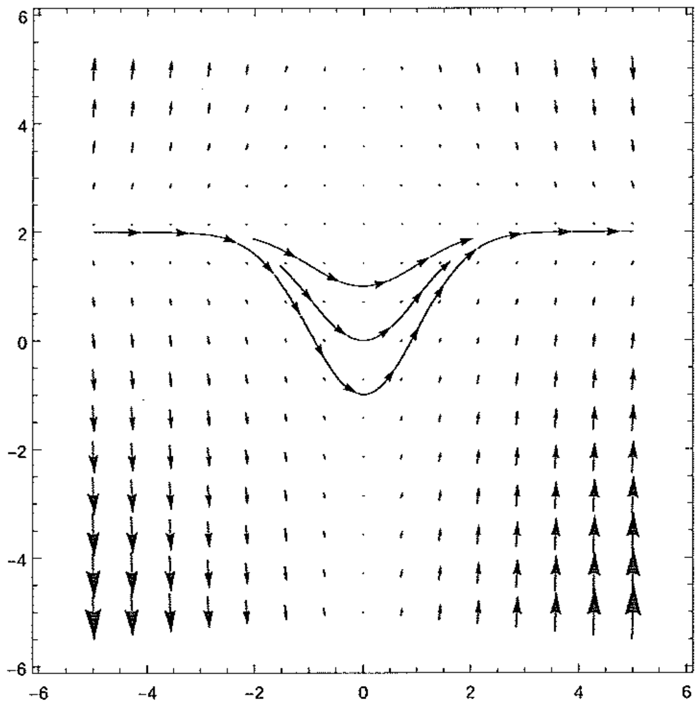
<u>Isoclines</u> are regions with the same slope.

<u>Integral curve</u> is just a fancy word for "solution function".

2) In[19]:=

```
VectorPlot[{1, x (2 - y)}, {x, -5, 5}, {y, -5, 5},
  StreamPoints → {{{{0, -1}, Black}, {{0, 0}, Black}, {{0, 1}, Black}}}]
```

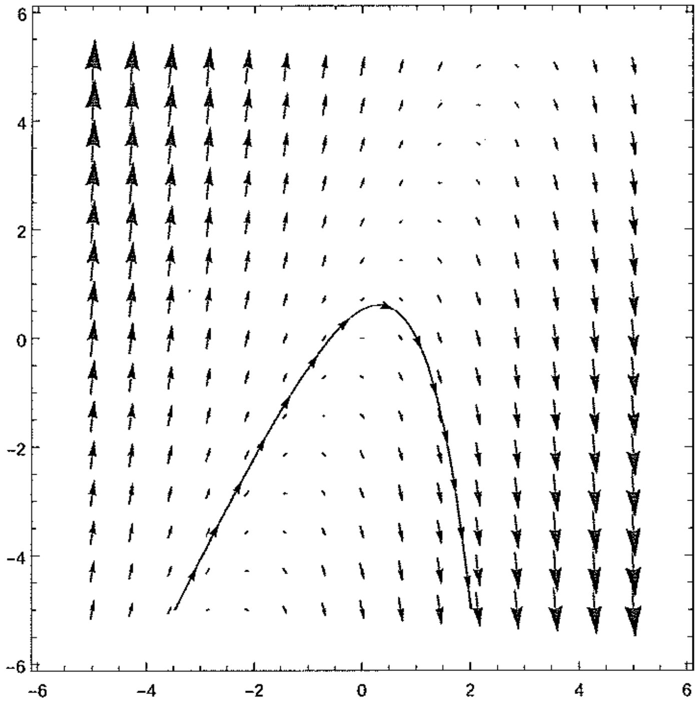
Out[19]=



3a)

In[12]:=
```
VectorPlot[{1, y - 2 x}, {x, -5, 5}, {y, -5, 5}, StreamPoints → {{{{1, 0}, Black}}}]
```

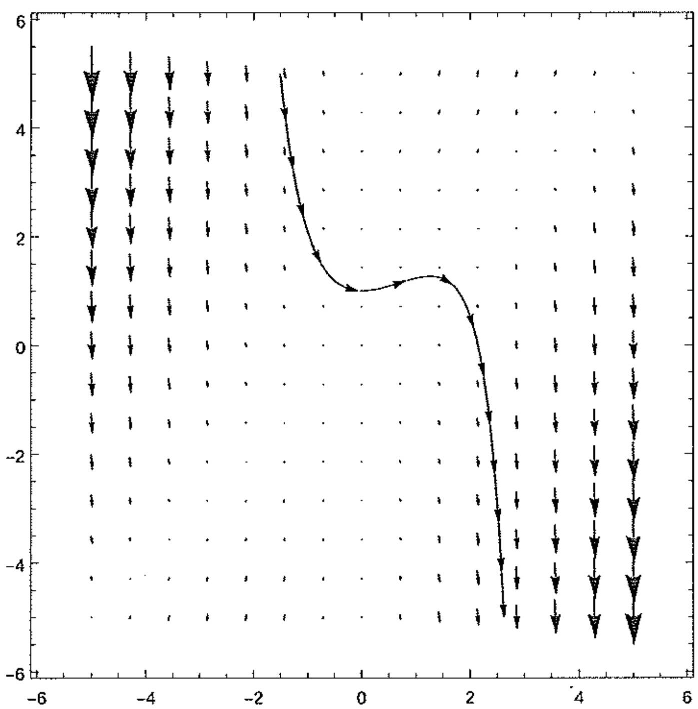
Out[12]=



b)

In[13]:=
```
VectorPlot[{1, x y - x^2}, {x, -5, 5}, {y, -5, 5}, StreamPoints → {{{{0, 1}, Black}}}]
```

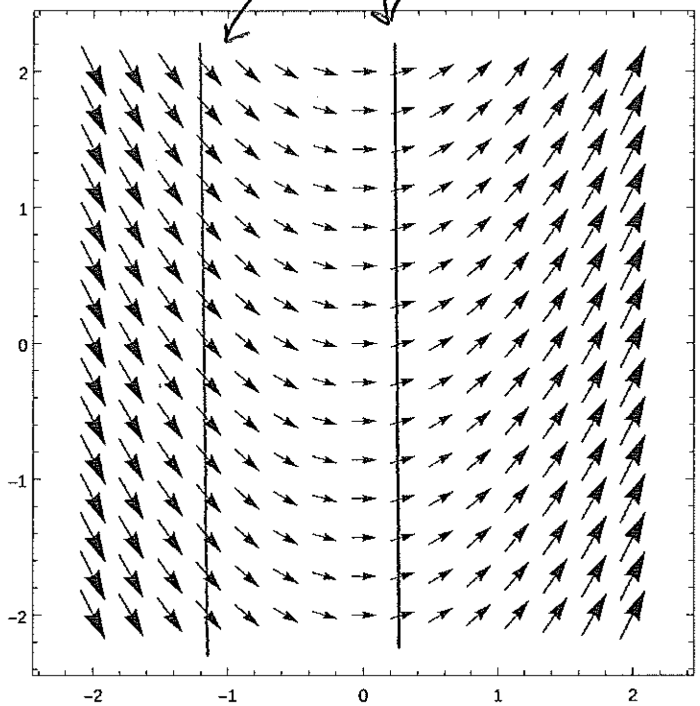
Out[13]=



4) isoclines

In[18]:= `VectorPlot[{1, t}, {t, -2, 2}, {y, -2, 2}]`

Out[18]=



In[17]:= `VectorPlot[{1, t}, {t, -2, 2}, {y, -2, 2},`
`StreamPoints → {{{{0, 1}, Black}, {{0, -1}, Black}}}]`

Out[17]=