MA 114 Worksheet # 29: The Logistic Equation

1. The population of the world in 1990 was around 5.3 billion. Assume the growth constant is 1/265 and the carrying capacity is 100 billion.

   (a) Write out the logistical model and solve it.

   (b) Use this to estimate the population in 2000 and compare it with the actual population of 6.1 billion.

   (c) Use the logistical model to predict the population in 2100 and 2500.

2. Assume the carrying capacity of the U.S. population is 5 billion.

   (a) Use this and the fact that the population in 1990 was 250 million to find the logistical model for the U.S. population. (Do not solve for k).

   (b) Use the fact that the population in 2000 was 275 million to find k and $P(t)$.

   (c) Predict the U.S. population in 2100 and 2500

   (d) When will the U.S. population reach 350 million?

3. A lake with a carrying capacity of 10,000 fish is stocked with 400 fish. The number of fish triples in the first year.

   (a) Find the logistical model and solve it. (Also find k).

   (b) How long will it take for the population to reach 5000 fish?

4. Let $c > 0$. A differential equation of the form

$$\frac{dy}{dt} = ky^{1+c}$$

   where $k > 0$ is called a *doomsday equation* because $1 + c > 1$.

   (a) Use separation of variables to find the solution of this model with $y(0) = y_0$.

   (b) Show that there is a finite time $t = T$ (doomsday) such that

$$\lim_{t \to T^-} y(t) = \infty.$$

   (c) A certain breed of rabbits has the growth rate term $ky^{1.01}$. If the initial population is 2 and there are 16 rabbits after 3 months, then when is doomsday?

Direction Fields and Euler's method:

1. Draw the direction field for the differential equation

$$y' = y + x.$$

   Sketch the solution which satisfies $y(0) = 0$.

2. Consider the differential equation

$$y' = x^2 - y.$$

   If at the n-th iteration of Euler's method with $h = .1$ we have $(x_n, y_n) = (3.1, -2)$, what is (x_{n+1}, y_{n+1})?