Worksheet 2 Key

1. a) A partial sum of the sequence $\{a_n\}$ is a finite sum of the first N terms, $S_N = \sum_{n=1}^{N} a_n$. A series converges if the sequence of partial sums converges.

   b) A sequence is a list of numbers. A series is an infinite sum.

   c) $a_n = \sin(n)$, $a_n = (-1)^n$

   d) No, but it must converge at or below M.

   e) The index is shifted by 1, but it is the same definition. Converges for $|r| < 1$, and diverges otherwise. If the series converges, it converges to $\frac{a}{1-r}$.

2. i) by rule 2

   ii) $\frac{2n^2}{2n+1} \overset{2}{\le} \frac{2n^2}{2n} = n$

   iii) $\frac{2n-1}{3n+2} \overset{1}{\le} \frac{2n}{3n+2} \overset{2}{\le} \frac{2n}{3n} = \frac{2}{3}$

   iv) $\frac{n+1}{2n^3+4} \overset{1}{\le} \frac{2n}{2n^3+4} \le \frac{2n}{2n^3} = \frac{1}{n^2}$

3. As the denominator increases, the terms become smaller. Always, $0 < \frac{1}{n} \le 1$. The sequence is monotone decreasing and bounded below, so it must converge.

4. a) If $a_n = f(n)$ for all n, then $f'(x) > 0$ implies a_n is strictly increasing.

   b) $f(x) = \frac{x}{x^2+1}$. $f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$.

   For $x > 1$, f is decreasing, so a_n is too.

   c) a_n is monotone decreasing, but always positive, so it converges.

5. a) bounded in $[1,2]$. Converges to 1. Monotone dec.

b) bounded in $[-1,1]$. Converges to 0.

c) Put $f(x) = \dfrac{x}{e^x}$. Then $f'(x) = \dfrac{e^x - xe^x}{(e^x)^2} = \dfrac{1-x}{e^x}$.

Decreasing for $x > 1$. Bounded below by 0. Converges.

6. a) $\displaystyle\sum_{n=1}^{\infty} \left(\tfrac{2}{5}\right)^n = \sum_{n=1}^{\infty} \tfrac{2}{5}\left(\tfrac{2}{5}\right)^{n-1} = \frac{2/5}{1-2/5} = \frac{2}{3}$

b) $\displaystyle\sum_{n=1}^{\infty} \left(-3/2\right)^{n-1}$ diverges, because $|-\tfrac{3}{2}| \geq 1$.

c) $\displaystyle 5 - \frac{5}{4} + \frac{5}{4^2} - \frac{5}{4^3} + \dots = \sum_{n=0}^{\infty} 5\left(-\frac{1}{4}\right)^n$

$$= \frac{5}{1-(-\frac{1}{4})} = 4$$

7. $\displaystyle\sum_{n=1}^{\infty} 4(.01)^n = \sum_{n=1}^{\infty} .04(.01)^{n-1}$

$$= \frac{.04}{1-.01} = \frac{.04}{.99} = \frac{4}{99}$$