MA 114 Worksheet # 3: More Sequences and Series

1. Conceptual Understanding:

   (a) For what values of x does the sequence $\{x^n\}$ converge.

   (b) For what values of x does the sequence $\{n^x\}$ converge.

   (c) If $\lim_{n\to\infty} b_n = \sqrt{2}$, find $\lim_{n\to\infty} b_{n-3}$.

   (d) Suppose you know that $\sum_{n=1}^{\infty} a_n = \pi$. Let S_n be the n-th partial sum of $\{a_n\}$. What can you say about the sequence $\{S_n\}$? What is the limit of the sequence $\{a_n\}$?

   (e) Explain the test for divergence. Does this test ever apply to prove that a series converges?

2. Your classmate claims that

$$\sum_{n=1}^{\infty} \frac{42}{n+1}$$

   converges since

$$\lim_{n\to\infty} \frac{42}{n+1} = 0.$$

   How do you respond?

3. Identify the following statements as true or false and explain your answers.

   (a) The sequences $\{a_1, a_2, \ldots\}$ and $\{a_1, a_2, \ldots, a_n\}$ are the same.

   (b) The sequence $\{a_n\}$ converges to A exactly when $\lim_{n\to\infty} a_n = A$.

   (c) If $\lim_{n\to\infty} |a_n|$ exists then $\lim_{n\to\infty} a_n$ exists.

   (d) If the sequence of partial sums of an infinite series is bounded the series converges.

   (e) $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} a_n$ if the series converges.

   (f) $\sum_{n=1}^{\infty} a_n = \sum_{n=0}^{\infty} a_{n+1}$ if both of the series converge.

   (g) If c is a nonzero constant and if $\sum_{n=1}^{\infty} c\, a_n$ converges than so does $\sum_{n=1}^{\infty} a_n$.

   (h) A finite number of terms of an infinite series may be changed without affecting whether or not the series converges.

   (i) Every infinite series with only finitely many nonzero terms converges.

4. Let a and r be constants with $-1 < r < 1$. Use the method of telescopic series (on p. 550) and the identity $ar^{n-1}(1-r) = ar^{n-1} - ar^n$ to find the sum of the series $\sum_{n=1}^{\infty} ar^n(1-r)$. Then apply the last equation of Theorem 1 on p. 551 to obtain the formula for the sum of geometric series.

5. Use the method of telescopic series and the identity $\ln(1+\frac{1}{n}) = \ln(n+1) - \ln(n)$ to determine whether or not the series $\sum_{n=1}^{\infty} \ln(1+\frac{1}{n})$ converges.

6. For what values of x is the function

$$f(x) = \sum_{k=1}^{\infty} (x-3)^k$$

   defined? Find a simple expression for $f(x)$ for these values of x. Is your expression defined for values of x where $f(x)$ is not defined?