MA 114 Worksheet # 4: Test for Divergence; Comparison Tests

1. Conceptual Understanding:

   (a) Explain the test for divergence. Why should you never use this test to prove that a series converges?

   (b) State the comparison test for series. Explain the intuition behind this test.

   (c) Suppose that the sequences $\{x_n\}$ and $\{y_n\}$ satisfy $0 \leq x_n \leq y_n$ for all n and that $\sum_{n=1}^{\infty} y_n$ is convergent. What can you conclude? What can you conclude if instead $\sum_{n=1}^{\infty} y_n$ diverges?

   (d) Find the error(s) in the following (incorrect) argument: The series $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ have all positive terms. The series $\sum_{n=1}^{\infty} 1/n^2$ is convergent and also for all n

   $$\frac{1}{n^2} \leq \frac{1}{n}.$$

   Therefore the series $\sum_{n=1}^{\infty} \frac{1}{n}$ must also converge by the comparison test.

   (e) State the limit comparison test. Explain how you apply this test.

2. A p-series is a series of the form

   $$\sum_{n=1}^{\infty} \frac{1}{n^p}.$$

   Under what conditions does this series converge? Under what conditions does this series diverge?

3. Write down the general form for a geometric series. Under what conditions does such a series converge? Under what conditions does such a series diverge? Contrast the geometric series with the p-series (that is, indicate how the two types of series are very different in form).

4. Determine if the following series converge or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{10}{n}\right)^{10}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt{n^2+2}}$

   (d) $\displaystyle\sum_{m=1}^{\infty} \frac{m^2+m+1}{3m^2+14m+7}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{2+5^n}$

   (f) $\displaystyle\sum_{k=1}^{\infty} \frac{2}{k^2+4k+3}$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

   (h) $\displaystyle\sum_{j=1}^{\infty} \frac{j}{j^2-\cos^2(j)}$