MA 114 Worksheet # 5: The Alternating Series Test

1. Conceptual Understanding:

   (a) State the alternating series test.

   (b) Prove that the alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

   (c) State the alternating series estimation theorem.

2. Test the following series for convergence or divergence.

   (a) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+2n}$

   (b) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$

   (c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{2/3}}$

   (d) $\sum_{n=1}^{\infty} \frac{3^n}{4^n + 5^n}$

   (e) $\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$

   (f) $\sum_{n=1}^{\infty} \left(\frac{-5}{17}\right)^{n-1}$

3. In practice it is often quite difficult to compute the exact value of the sum of an infinite series. In applications however, this is rarely a problem as answers to "real world" problems typically need only be accurate up to a certain threshold called an *error tolerance*. Suppose you need to compute the sum of the alternating harmonic series with an error tolerance of 10^{-6}. How many terms should you take in your partial sum to approximate the value of the sum with this error tolerance? Write down (but do not compute) the partial sum which approximates the sum of the series up to the error tolerance. Based on your results, comment on the speed of convergence for the alternating series.

4. Prove that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{10^n n!}$ converges and determine the number of terms you need to approximate the sum up to an error tolerance of .0001. Write down (but do not compute) a partial sum which approximates the sum of the series. Based on your results, comment on the speed of convergence of the series.

5. Use a partial sum to approximate $\sum_{n=1}^{\infty} \left(\frac{-1}{5}\right)^{n-1}$ accurate to 3 decimal places. What is the exact value of the sum? Verify that the error is of the correct size.