Worksheet 5 Key

1. a) Suppose $\{a_n\}$ is a decreasing sequence converging to zero. Then $\sum (-1)^n a_n$ converges.

b) Since $\frac{1}{n} \to 0$, the alternating harmonic series converges.

c) If $S = \sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges, then
$$|S - S_N| < a_{N+1}.$$
That is, the error is less than the first omitted term.

2. a) Converges by AST

b) Converges by AST

c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{2/3}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$ converges by AST

d) $\sum_{n=1}^{\infty} \frac{3^n}{4^n+5^n} \leq \sum_{n=1}^{\infty} \frac{3^n}{4^n+4^n} = \frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$

converges by comparison to geometric series.

e) Let $a_n = \frac{n}{\ln(n)}$, $f(x) = \frac{x}{\ln(x)}$.

$\lim_{x\to\infty} \frac{x}{\ln(x)} \overset{LH}{=} \lim_{x\to\infty} \frac{1}{1/x} = \infty$. By test for divergence, this diverges.

f) $\sum_{n=1}^{\infty} \left(\frac{-5}{17}\right)^{n-1}$ converges by geometric series with $|r| < 1$.

3. If we want the error to be $\leq 10^{-6}$, then we must go out to the $10^6 - 1$ st term. Then by error bound equation, $|S - S_{10^6-1}| \leq \frac{1}{10^6}$. $S_{10^6-1} = \sum_{n=1}^{10^6-1} \frac{1}{n}$. The series converges rather slowly. To guarantee at least 6 correct decimal places, the first 10^6-1 terms must be computed.

4. The series clearly converges by AST,

$a_3 = .0001\overline{6}$

$a_4 = .00000041\overline{6}$

$|S - S_3| < a_4 < .0001$, so we only need 3 terms to achieve the desired precision. This series converges very quickly.

5. $\sum_{n=1}^{\infty} \left(-\frac{1}{5}\right)^{n-1} = \sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{5}\right)^n = \sum_{n=0}^{\infty} (-1)^n a_n$

where $a_n = \left(\frac{1}{5}\right)^n$. To ensure error less than $.001$, check terms: $a_4 = .0016$, $a_5 = .00032$. So use $S_4 = 1 - 1/5 + 1/25 - 1/125 + 1/625$

$= .8336$, because

$|S - S_4| < .00032$.

The exact value of the series is $\frac{1}{1-(-\frac{1}{5})} = \frac{5}{6}$,

which is approximately $.83\overline{3}$. So we were correct to 3 decimal places.