MA 114 Worksheet # 6: Testing for Convergence of a Series

1. Conceptual understanding:
   (a) Let $a_n = \dfrac{n}{3n+1}$. Does $\{a_n\}$ converge? Does $\sum_{n=1}^{\infty} a_n$ converge?
   (b) Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ where $\lim_{n\to\infty} a_n = 0$.
   (c) Does there exist a convergent series $\sum_{n=1}^{\infty} a_n$ which satisfies $\lim_{n\to\infty} a_n \neq 0$? Explain.
   (d) When does a series converge absolutely? When does a series converge conditionally?
   (e) State the root and ratio tests.

2. Use any valid convergence/divergence test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.
   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[5]{n}}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n^2}{n!}$

3. Test the following series for convergence or divergence. Clearly state what series test you apply and demonstrate that all the hypotheses for that test are satisfied.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^5+1}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{(11-\cos^2(n))^n}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{\sqrt{2n^2+2}}$
   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$
   (e) $\displaystyle\sum_{n=1}^{\infty} 13\cos(5)^{n-1}$
   (f) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{3n^3+2n}{4n^3+1}\right)^n$
   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$

4. Identify the following statements as true or false. If the statement is true, cite evidence from the text to support it. If the statement is false, correct it so that it is a true statement from the text.
   (a) To prove that the series $\sum_{n=1}^{\infty} a_n$ converges you should compute the limit $\lim\limits_{n\to\infty} a_n$. If this limit is 0 then the series converges.
   (b) To apply the ratio test to the series $\sum_{n=1}^{\infty} a_n$ you should compute $\lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|}$. If this limit is less than 1 then the series converges absolutely.
   (c) To apply the root test to the series $\sum_{n=1}^{\infty} a_n$ you should compute $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|}$. It this limit is 1 or larger then the series diverges.
   (d) One way to prove that a series is convergent is to prove that it is absolutely convergent.
   (e) An infinite series converges when the limit of the sequence of partial sums converges.