

MA 114 Worksheet # 7: Power Series

1. Conceptual Understanding:

(a) For what values of x does the series $\sum_{k=1}^{\infty} 2 \cos(x)^{k-1}$ converge?

(b) Find a formula for the coefficients c_i of the power series

$$\frac{1}{0!} + \frac{2}{1!}x + \frac{3}{2!}x^2 + \frac{4}{3!}x^3 + \dots$$

(c) Find a formula for the coefficients c_i of the power series

$$1 + 2x + x^2 + 2x^3 + x^4 + \dots$$

(d) Suppose $\lim_{n \rightarrow \infty} \sqrt[n]{|c_n|} = c$ where $c \neq 0$. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$.

(e) Consider the function $f(x) = \frac{5}{1-x}$. Find a power series that is equal to $f(x)$ for every $|x| < 1$.

(f) Define the terms *power series*, *radius of convergence*, and *interval of convergence*.

2. Find the radius and interval of convergence for the following power series.

(a) $\sum_{n=0}^{\infty} (5x)^n$

(b) $\sum_{n=1}^{\infty} \sqrt{n} x^n$

(c) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(d) $\sum_{n=2}^{\infty} \frac{x^n}{3^n \ln n}$

(e) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^4}$

(g) $\sum_{n=1}^{\infty} \frac{(5x)^n}{n^3}$

3. Suppose that the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is R . What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{2n}$?

4. Let

$$f(r) = \frac{1}{1-r}$$

(a) Find a power series expansion for $f(r)$. What is the radius and interval of convergence for this series?

(b) Let $r = -x$ in $f(r)$ and determine the corresponding power series expansion. What is the radius and interval of convergence?

(c) Let $r = x^2$ in $f(r)$ and determine the corresponding power series expansion. What is the radius and interval of convergence?

(d) Let $r = \frac{x^2}{9}$ in the series expansion for $f(r)$. For what values of x does this series converge?