Worksheet 7 Key

1. a) Converges for $|\cos(x)| < 1$, so just avoid multiples of π. That is, $x \neq n\pi$ for any integer n.

b) $c_i = \frac{n+1}{n!}$

c) $1.5 + (-1)^{n+1}(.5) = c_i$

d) Root test: $\lim_{n\to\infty} \sqrt[n]{|c_n x^n|} = c|x| < 1$

$$|x| < 1/c = R$$

$$-1/c < x < 1/c$$

e) $\sum_{n=0}^{\infty} 5x^n$

f) See Rogawski section 10.6, page 579.

2. a) Converges for $|5x| < 1 \Rightarrow |x| < 1/5$.

$R = 1/5$, interval $(-1/5, 1/5)$.

b) Ratio test: $\lim_{n\to\infty} \left| \frac{\sqrt{n+1}\, x^{n+1}}{\sqrt{n}\, x^n} \right| = \lim_{n\to\infty} |x| < 1 = R$.

Interval $(-1, 1)$.

c) Ratio test: $\lim_{n\to\infty} \left| \frac{x^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{x^n} \right| = |x| < 1 = R$

Interval $[-1, 1)$.

d) Ratio test: $\lim_{n\to\infty} \left| \frac{x^{n+1}}{3^{n+1}\ln(n+1)} \cdot \frac{3^n \ln(n)}{x^n} \right| = \left| \frac{x}{3} \right| < 1$

$R = 3$, interval $[-3, 3)$.

e) Root test: $\lim_{n\to\infty} \sqrt[n]{\left| \frac{x-2}{n} \right|^n} = \lim_{n\to\infty} \left| \frac{x-2}{n} \right| = 0$

$R = \infty$, interval $(-\infty, \infty)$.

f) Ratio test: $\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^4} \cdot \frac{n^4}{x^n} \right| = |x| < 1$

$R = 1$, interval $[-1, 1]$.

g) Ratio test: $\lim_{n\to\infty} \left| \frac{(5x)^{n+1}}{(n+1)^3} \cdot \frac{n^3}{(5x)^n} \right| = |5x| < 1$

$R = 1/5$, Interval $[-1/5, 1/5]$

3. Ratio test: $\lim_{n\to\infty} \left|\frac{c_{n+1} x^{n+1}}{c_n x^n}\right| = \lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| |x| < 1$

$$|x| < \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right| = R.$$

Ratio test: $\lim_{n\to\infty} \left|\frac{c_{n+1} x^{2n+2}}{c_n x^n}\right| = \frac{1}{R} |x^2| < 1$

$$x < \sqrt{R}.$$

4. a) $f(r) = \sum_{n=0}^{\infty} r^n$. Radius $R=1$. Interval $(-1,1)$.

b) $f(r) = f(-x) = \sum_{n=0}^{\infty} (-1)^n x^n$. Same radius/interval.

c) $f(r) = f(x^2) = \sum_{n=0}^{\infty} x^{2n}$, Need $|x^2| < 1$. Same radius/interval.

d) $f(r) = f\left(\frac{x^2}{9}\right)$

$= \sum_{n=0}^{\infty} \left(\frac{x^2}{9}\right)^n$ converges for $\left|\frac{x^2}{9}\right| < 1$

$$x^2 < 9$$

$$|x| < 3.$$

Radius $R=3$. Interval $(-3,3)$.