

MA 114 Worksheet # 9: Taylor and Maclaurin Series

1. Conceptual Understanding:

- (a) Suppose that $f(x)$ has a power series representation for $|x| < R$. What is the general formula for the Maclaurin series for f ?
- (b) Suppose that $f(x)$ has a power series representation for $|x - a| < R$. What is the general formula for the Taylor series for f about a ?
- (c) Let $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$. Find the Maclaurin series for f .
- (d) Let $f(x) = 1 + 2x + 3x^2 + 4x^3$. Find the Taylor series for $f(x)$ about $x = 1$.

2. Deduce the Maclaurin series for $\cos(x)$ from the Maclaurin series for $\sin(x)$ by applying Theorem 2 on p. 584. Prove that $\cos(x)$ is equal to the sum of its Taylor expansion for all x .

3. Assume that each of the following functions has a power series expansion. Find the Maclaurin series for each. Be sure to provide the domain on which the expansion is valid.

- (a) $f(x) = \ln(1 + x)$
- (b) $f(x) = xe^{2x}$

4. Use a known Maclaurin series to obtain the Maclaurin series for the given function. Specify the radius of convergence for the series.

- (a) $f(x) = \frac{x^2}{1 - 3x}$
- (b) $f(x) = e^x + e^{-x}$
- (c) $f(x) = xe^{2x}$
- (d) $f(x) = e^{-x^2}$
- (e) $f(x) = x^5 \sin(3x^2)$
- (f) $f(x) = \sin^2(x)$ Hint: $\sin^2(x) = (1 - \cos(2x))/2$