

MA 114 Worksheet #14: Power Series

1. (a) Give the definition of the radius of convergence of a power series $\sum_{n=0}^{\infty} a_n x^n$
- (b) For what values of x does the series $\sum_{n=1}^{\infty} 2(\cos(x))^{n-1}$ converge?
- (c) Find a formula for the coefficients c_k of the power series $\frac{1}{0!} + \frac{2}{1!}x + \frac{3}{2!}x^2 + \frac{4}{3!}x^3 + \dots$.
- (d) Find a formula for the coefficients c_n of the power series $1 + 2x + x^2 + 2x^3 + x^4 + 2x^5 + x^6 + \dots$.
- (e) Suppose $\lim_{n \rightarrow \infty} \sqrt[n]{|c_n|} = c$ where $c \neq 0$. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$.
- (f) Consider the function $f(x) = \frac{5}{1-x}$. Find a power series that is equal to $f(x)$ for every x satisfying $|x| < 1$.
- (g) Define the terms *power series*, *radius of convergence*, and *interval of convergence*.

2. Find the radius and interval of convergence for

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n n}{4^n} (x-3)^n$.

(g) $\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}}$

(b) $4 \sum_{n=0}^{\infty} \frac{2^n}{n} (4x-8)^n$.

(h) $\sum_{n=0}^{\infty} \frac{x^n}{3^n \ln n}$

(c) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(-3)^n}$.

(i) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^n}$

(d) $\sum_{n=0}^{\infty} n! (x-2)^n$.

(j) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^4}$

(e) $\sum_{n=0}^{\infty} (5x)^n$

(k) $\sum_{n=0}^{\infty} \frac{(5x)^n}{n^3}$

(f) $\sum_{n=0}^{\infty} \sqrt{n} x^n$

3. Use term-by-term integration and the fact that $\int \frac{1}{1+x^2} dx = \arctan(x)$ to derive a power series centered at $x = 0$ for the arctangent function. HINT: $\frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$.
4. Use the same idea as above to give a series expression for $\ln(1+x)$, given that $\frac{dx}{1+x} = \ln(1+x)$. You will again want to manipulate the fraction $\frac{1}{1+x} = \frac{1}{1-(-x)}$ as above.
5. Write $(1+x^2)^{-2}$ as a power series. HINT: use term-by-term differentiation.