MA 114 Worksheet #15: Taylor and Maclaurin Series

1. (a) Suppose that $f(x)$ has a power series representation for $|x| < R$. What is the general formula for the Maclaurin series for f?
   (b) Suppose that $f(x)$ has a power series representation for $|x - a| < R$. What is the general formula for the Taylor series for f about a?
   (c) Let $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$. Find the Maclaurin series for f.
   (d) Let $f(x) = 1 + 2x + 3x^2 + 4x^3$. Find the Taylor series for $f(x)$ centered at $x = 1$.

2. Assume that each of the following functions has a power series expansion. Find the Maclaurin series for each. Be sure to provide the domain on which the expansion is valid.
   (a) $f(x) = \ln(1 + x)$
   (b) $f(x) = xe^{2x}$

3. Use a known Maclaurin series to obtain the Maclaurin series for the given function. Specify the radius of convergence for the series.
   (a) $f(x) = \dfrac{x^2}{1 - 3x}$
   (b) $f(x) = e^x + e^{-x}$
   (c) $f(x) = e^{-x^2}$
   (d) $f(x) = x^5 \sin(3x^2)$
   (e) $f(x) = \sin^2 x$.
   HINT: $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$

4. Find the following Taylor expansions about $x = a$ for each of the following functions and their associated radii of convergence.
   (a) $f(x) = e^{5x}$, $a = 0$.
   (b) $f(x) = \sin(\pi x)$, $a = 1$.

5. Differentiate the series

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

   to find a Taylor series for $\cos(x)$.

6. Use Maclaurin series to find the following limit: $\displaystyle\lim_{x\to 0} \frac{x - \tan^{-1}(x)}{x^3}$.

7. Approximate the following integral using a 6th order Taylor polynomial for $\cos(x)$: $\displaystyle\int_0^1 x\cos(x^3)\,dx$

8. Use power series multiplication to find the first three terms of the Maclaurin series for $f(x) = e^x \ln(1 - x)$.