MA 114 Worksheet #16: Review for Exam 02

1. List the first five terms of the sequence:

   (a) $a_n = \dfrac{(-1)^n n}{n!+1}$ (b) $a_1 = 6,\ a_{n+1} = \dfrac{a_n}{n}$.

2. Determine whether the sequence converges or diverges. If it converges, find the limit.

   (a) $a_n = 3^n 7^{-n}$

   (b) $a_n = \dfrac{(-1)^{n+1} n}{n+\sqrt{n}}$

   (c) $a_n = \dfrac{\ln n}{\ln 2n}$

   (d) $a_n = \dfrac{\cos^2 n}{2^n}$

3. Explain what it means to say that $\sum_{n=1}^{\infty} a_n = 2$.

4. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^{n-1}}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{6\cdot 2^{n-1}}{3^n}$

5. Determine whether the given series converges or diverges and state which test you used.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{7\sqrt{n}}{5n^{3/2}+3n-2}$

   (c) $\displaystyle\sum_{n=1}^{\infty} n!e^{-8n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\ln n}{5n+7}\right)^n$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{9^n}{9n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$

   (g) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$

6. Determine whether the given series is absolutely convergent or conditionally convergent or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n^2}\right)$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$

7. Find the radius and interval of convergence of the series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$

8. Find a power series representation for the function and determine its radius of convergence.

   (a) $f(x) = \dfrac{5}{1-4x^2}$

   (b) $f(x) = \dfrac{x^2}{x^4+16}$

   (c) $f(x) = \dfrac{3}{2+2x}$

   (d) $f(x) = e^{-x^2}$

9. Using the formula

$$\ln(1+x) = \int_0^x \frac{1}{1+t}\,dt$$

   find a power series for $\ln(1+x)$ and state its radius of convergence.

10. Use the Maclaurin series for $\cos(x)$ to compute

$$\lim_{x\to 0} \frac{1-\cos(x)}{x^2}.$$